Integrated Math II
Instructional Focus Documents

Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 4: Performance at this level demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in this same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

The Real Number System (N.RN)

Standard M2.N.RN.A.1 (Major Work of the Grade)
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Scope and Clarifications:
For example, we define $5^{1/3}$to be the cube root of 5 because we want $\left(5^{1/3}\right)^3 = 5^{(1/3)3}$to hold, so $\left(5^{1/3}\right)^3$ must equal 5.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify expressions with rational exponents. Match equivalent forms of expressions involving integer exponents that include the product of powers or power of powers. Match equivalent forms of expressions involving integer exponents that include the quotient of powers..	Identify patterns that arise from properties of integer exponents and connect them to rational exponents. Rewrite the expression in radical form, given an expression with a rational exponent. Recognize the relationship between square root and an exponent of 1/2 and a cubed root and an exponent of 1/3.	Explain the relationship between the rational exponent, the index of the radical, and the power of the expression. Explain the difference between rewriting equivalent expressions by taking the square root of a number and solving an equation which includes a square root, using the principal square root function. Compare properties of integer exponents with properties of rational exponents.	Construct a viable argument explaining the relationship between rational exponents and radical notation. Critique the reasoning of others by finding errors and justify changes that could be made to correct mistakes.

Instructional Focus Statements

Level 3:

In Integrated math I, standard M1.A.SSE.B.2a, students experienced using the power of a power, power of a product, and quotient of powers properties with integer exponents. In integrated math II, students extend their knowledge of these to include rational exponents. Instruction should include problems where students see a connection between the inverse operations of multiplication and division and how these inverse operations are expanded to radical and exponential forms of numbers. Instruction should begin by giving students the opportunity to explore examples such as $\left(\sqrt{16}\right)^2$ to make the connection that this is the same thing as 4^2. By exploring this concept, students will understand how the square of a number is the inverse operation of taking the square root. Instruction should also lead students to the understanding that the ½ power represents a square root. Students should be expected to rewrite expression using factorizations and exponent rules to apply the meaning of these rational exponents. For example, given $\left(\sqrt{16}\right)^3$, students will rewrite the example as $16^{2/3}$. This would help students recognize the square root of 16 is equal to 4, and then cube that number to reveal the answer of 64. Multiple solution methods could be explored to help students find what patterns hold true to reinforce the order of operations and various exponent properties.

A common misconception for students is the confusion of the meaning of an exponent -2 with the meaning of ½ power. Students may confuse $(9^{-2})^2$ as the same as $\left(9^{1/2}\right)^2$. In addition, students often confuse rewriting expression in a simpler form by taking the square root of a number with finding the solution set of an equation. Discussion should address the difference between the $\sqrt{16}$ and $x^2 = 4$, using the principal square root function. With full understanding of this standard, students should be able to explain why $\left(4^{1/2}\right)^2 = 4$.

Level 4:

To deepen the level of understanding, instruction should provide the opportunity for students to engage in more complex problems involving rational exponents and the notation for radicals in terms of rational exponents. Students will apply properties of rational exponents in order to simplify an expression and change parts written in radical form into exponential form. This process might make it easier for students to apply properties of exponents. An example would be to simplify $\sqrt[3]{x^2} * \left(\sqrt[6]{x^4}\right)^{-3}$ or $\frac{\sqrt[3]{x} * \sqrt{x^5}}{\sqrt{25x^{16}}}$. As students simplify these expressions, they should justify their steps and explain their reasoning.

Additionally, focus should be placed on critiquing the reasoning of others. Instructions should include providing students with expressions that include rational exponents and radicals which have been simplified. Students determine if the expressions are simplified correctly and if not, they should correct the mistakes and explain why it was incorrect.

Standard M2.N.RN.A.2 (Major Work of the Grade)
Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the index and power of an expression written in exponential form. Identify the index and power of an expression written in radical form.	Match expressions written in radical form with the equivalent expression written in exponential form. Match expressions written in exponential form with the equivalent expression written in radical form.	Write an equivalent expression using rational exponents, given an expression in radical form. Write an equivalent expression using radicals, given an expression written exponential form. Move fluently between radical and exponential form of expressions. Explain the process of changing an expression from radial form to exponential form. Explain the process of changing an expression from exponential form to radical form. Write an equivalent expression using the properties of exponents, given an expression with rational exponents.	Simplify complex fractions which contain radicals in the denominator. Critique the reasoning of others, given expression which contains rational exponents and/or radicals and the steps to simplify it. Construct an argument explaining why or why not a chosen strategy to simplify expressions best suits the problem.

Instructional Focus Statements

Level 3:

Students should be able to simplify fluently between radical and exponential form. Instruction should expose students to a variety of problems that include radicals and exponents so that they realize that some problems are easier to work a certain way. Students should be provided expressions that can be simplified in either radical or exponential form. By having students rely on their knowledge of exponent rules and meaning of radicals, by asking students to solve without direct steps, students may present multiple ways to simplify these expressions. Discussion can then take place on different methods that can work on the same type of problem and allow teachers the chance to explain there is not only one correct method. After students have practiced with multiple approaches, they should be able to choose which approach they prefer and defend their reasoning for choosing a particular method.

Students should be given both symbolic and contextual problems to solidify their understanding of radicals and rational exponents. Symbolic examples could be problems such as $c^2 * \sqrt{c} = c^2 * c^{1/2}$. This will help students understand they must find a common denominator to add 2 and ½ to get a new power of 5/2. Students could also be given problems involving the volume of a sphere to apply these skills in contextual situations.

Level 4:

Exposing students to more complex expressions, such as ones that include fractions with radicals in the denominator, will help deepen students understanding of this standard. Integrated math II is the first math course where students are asked to rationalize denominators. Instruction should include providing students with simple problems where they are given a radical in the denominator. Challenging students to find another expression that, when multiplied, will produce a value that has an integer root will lead to discussions about repeated factors and various types of roots. For example, students should understand that when the $\sqrt[5]{4} * \sqrt[5]{8} = \sqrt[5]{32} = 2$ and would eliminate the radical.

As students become familiar with these types of expressions, instruction will extend to fractions and eventually, complex fractions. As students become proficient with rationalizing the denominator, they should understand why this process is important for eliminating the radical. Instruction should also include expressions where variables occur under the radical sign. Students should be introduced to the Absolute-Value-Square Root Theorem and in simple terms understand that if an odd power is removed from an even root, an absolute value sign will ensure variables have a positive value. In addition, students should be provided with radical and exponential expressions which have been simplified.

Quantities (N.Q)

Standard M2.N.Q.A.1 (Supporting Content)
Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.

Scope and Clarifications: (Modeling Standard)
Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.
Tasks are limited to linear, quadratic, exponential equations with integer exponents, square root, and cube root functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the units in a problem. Connect the units to the values in a real-world problem.	Identify individual quantities in context of the real-world problem and label them with appropriate units. Recognize irrelevant or extraneous information in a real-world problem.	Identify and interpret information to select or create a quantity to model a real-world problem. Describe individual quantities in context of the real-world problem. Attend to precision when defining quantities and their units in context. Explain and justify the relationship between a solution to a contextual problem and the values used to compute the solution. Appropriately interpret, explain the meaning of, and draw conclusions about the quantities in a real-world problems.	Identify, interpret, and justify complex information with a variety of descriptors or units to solve contextual problems for the purpose of descriptive modeling. Represent quantities in descriptive modeling situations and explain their relationship using multiple formats such as numeric, algebraic, and graphic representations.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Make observations about quantities given a graph or model. Interpret and explain irrelevant or extraneous information in a real-world problem.	

Instructional Focus

Level 3:

In grades K-8, students developed an understanding of measuring, labeling values, and understanding how the value of the number relates to the described quantity. In the high school NQ domain, students develop an understanding of reasoning quantitatively and using units to solve problems. This standard should be taught within integration with other standards throughout the course. Students should extend this understanding by applying their knowledge to modeling situations where they can make comparisons between two distinct quantities and justify the quantities appropriately in order to describe a contextual problem. Instruction should focus on providing opportunities of real-world problems where students have to select appropriate quantities and attend to precision in describing the quantities in descriptive modeling situations. Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc. The study of dimensional analysis is an excellent avenue to help students understand how critical values, units, and quantities are used in interpreting information and modeling a real-world problem. Furthermore, students must be given opportunities to write and create appropriate labels for quantities and explain the meaning of the quantities in a context. Being able to identify, interpret, and justify quantities is a skill that will serve students well to have mastered during this course as this standard lays the foundation for using units as a way to understand problems. In this course, students should sufficiently explore linear, quadratic, exponential equations with integer exponents, square root, and cube root functions.

Level 4:

Instruction should focus on providing opportunities for students to work with problems that have a variety of descriptors and units within the context. Students should be asked to extend their knowledge of quantities by representing them in multiple formats such as a graphical representation of the given information, algebraic representation of the quantities, and multiple representations to predict or draw conclusions about the solution of the real-world problem. Instruction should provide opportunities for students to analyze and critique the interpretation of quantities in a descriptive modeling problem. Additionally, students should be given ample opportunities that promote inquiry to design their own contextual problem in which they would have to use quantities appropriately in order to describe the modeled contextual situation.

Now that students have more exposure to multiple function types they should be given the opportunity to interpret and distinguish the difference in the quantities produced by different functions, such as square units or cubic units. Instruction should require students to explain why the units of a quantity change depending on the function by which it is represented.

The Complex Number System (N.CN)

Standard M2.N.CN.A.1 (Supporting Content)
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify when a square root represents an imaginary number. Rewrite the square root of a negative perfect square number as a pure imaginary number.	Rewrite a number containing only the square root of a negative number in a + bi form. Differentiate between the real and imaginary terms in a complex number.	State that there is a complex number *i* such that $i^2 = -1$. Distinguish between a real number, a pure imaginary number and a complex number. Express complex numbers in the written form $a + bi$, where a and b represent real numbers.	Recognize that solutions to problems may be complex numbers and identify whether or not the solutions are viable within a mathematical or real-world context. Plot complex numbers on a complex plane. Find the absolute value of a complex number graphed on a complex plane.

Instructional Focus

Level 3:

Integrated Math II is a student's first interaction with the complex number system. Students learn that the definition of an imaginary unit is $i = \sqrt{-1}$ and they learn that the solutions for $x^2 = -1$ are $\pm i$. A foundational focus for this standard is for student to understand why taking the square root of a negative number generates a complex number. Students should also develop an understanding that all numbers can be written in the form $a + bi$ when a and b are both real numbers. Students should be able to recognize pure real numbers in the form of $a + 0i$ and pure imaginary numbers in the form of $0 + bi$ to solidify the understanding that both pure real and pure imaginary numbers can be written in complex form. Additionally, students should be able to describe a situation from which complex numbers can emerge, such as a quadratic equation in the form of $x^2 = a$, where a is a negative real number.

Level 4:

As students develop a deep understanding of complex numbers, they will be able to construct a viable argument to describe why taking the square root of a negative number generates an imaginary number. Students should also solve equations from contexts eliciting complex numbers and be able to explain why the context generated a complex solution and whether the solution is viable or not viable. As they solidify their understanding, students will justify their reasoning as to why some solutions are viable while others are not in a real-world situation. To support the study of the magnitude of vectors in future courses, students can be challenged to plot complex numbers in the complex plane. Student thinking could then be pushed for students to apply what they know about the relationship between absolute value and distance to discover how to calculate the absolute value of complex numbers utilizing the complex plane. Students come to realize that when drawing a line from the complex number to the origin and creating a right triangle, they can use their knowledge of right triangles to find the magnitude (length) of the line which represents the absolute value of the complex number.

Standard M2.N.CN.A.2 (Supporting Content)
Know and use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the real and imaginary parts of a given complex number. Identify that all imaginary numbers are like terms and all real numbers are like terms.	Add, subtract, and multiply pure imaginary numbers.	Add, subtract, and multiply complex numbers. Explain when the commutative, associative, and distributive properties are helpful when writing equivalent expressions involving complex numbers. Recognize there is a pattern that will emerge when *i* is raised to positive integer powers.	Write an equivalent expression to a given multi-operational expression that involves adding, subtracting, and/or multiplying complex numbers, identify the properties used within the simplification, and provide justification for why the original and the new expression are equivalent. Explain how the commutative, associative, and distributive properties are helpful when rewriting equivalent expressions involving complex numbers. Identify the pattern that exists when *i* is raised to positive integer powers and explain why it occurs.

Instructional Focus Statements

Level 3:

The complex number system is first introduced to students in Algebra II. It is important that instruction for this standard be closely tied to the conceptual understanding of complex numbers developed in standard M2.N.CN.A.1. The focus of instruction for this standard should initially provide students opportunities to compare and contrast computations with complex numbers to computations with rational and irrational numbers. For example, students may consider expressions such as $(3-2\sqrt{5})+(7+8\sqrt{5})$ and $(3 - 2i) + (7 + 8i)$ noting the similarities and differences that exist between the two. It is important that students not be provided a list of rules and steps to follow when working with addition, subtraction, and multiplication of complex numbers. They need to be given time to develop an understanding of how and why complex numbers interact the way they do with the operations of addition, subtraction, and multiplication.

As students become more fluent in computing with complex numbers, they should begin thinking about how mathematical properties apply to simplifying expressions such as $5(6-i)$ and $3i\,(4-2i)$, when each property is useful, and why. Ultimately, students should self-identify processes for simplifying expressions containing complex numbers.

Students at this level may begin to recognize that there is a pattern existing when *i* is raised to positive integer powers as they work with simplifying increasingly challenging expression.

Students will use their knowledge of complex numbers in future courses when working with vectors.

Level 4:

At this level, students should be challenged to write an equivalent expression to a given multi-operational expression that involves adding, subtracting, and/or multiplying complex numbers. Students with a deep understanding of complex numbers and how each is effected by addition, subtraction, and multiplication should be able to seamlessly move between the operations when presented multi-operational expressions. Additionally, students should be able to identify the properties used within the simplification and provide justification for why the original and the new expression are equivalent.

Students at this level not only recognize that there is a pattern existing when *i* is raised to positive integer powers, but also explicitly identify the pattern and explain why it occurs. This identification should be the by-product of students noticing the pattern as they work to simplify increasingly challenging expressions.

Students could also be challenged to apply their knowledge of operations with complex numbers to solve real-world problems such as finding the impedance of a parallel circuit with two pathways giving students the opportunity to see applications of complex numbers in real world situations.

Standard M2.N.CN.B.3 (Supporting Content)
Solve quadratic equations with real coefficients that have complex solutions.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the real and imaginary part, given the solution to a quadratic equation. Determine if the roots are real or non-real, given the graph of a quadratic equation. Recognize if an equation in the form of $x^2 = r$ would have real or non-real solutions. Calculate the discriminant of a quadratic equation.	Use the discriminant to determine if solutions to a quadratic equation are real or non-real. Solve a simple quadratic equation using inverse operations (including taking the square root of both sides) and identify the solutions as real or complex.	Solve using the quadratic formula and identify solutions as real or complex, given a quadratic equation in the form $ax^2 + bx + c = 0$. Solve by completing the square and identify solutions as real or complex, given a quadratic equation in the form $ax^2 + bx + c = 0$. Explain why quadratic functions may produce real or complex solutions. Determine the reasonableness of solutions by graphing a quadratic function and examining the roots.	Solve the equation and determine if all solutions are viable in the context of the problem, given a real word situation that reflects a quadratic function. Explain why a quadratic equation has a complex solution based on the operations used to solve and the definition of $i^2 = -1$.

Instructional Focus Statements

Level 3:

In Integrated Math II, instruction should help students realize all solutions to a quadratic equation are not real numbers and that complex solutions can arise. Modeling with graphing tools will help students understand that solutions to a quadratic function may include one x-intercept, two x-intercepts, or no x-intercepts. They should connect a graph with no x-intercepts to an equation with complex roots. Discussion should also take place to help students distinguish between irrational roots and imaginary roots. Many times, students have difficulty understanding that an irrational root falls in the real number system. They assume because an irrational number does not have an exact value that it is a complex number. Students should be provided with quadratic equations which result in irrational roots and discussion should focus on comparing the location of these roots on a graph with complex roots. The connection to the graphical representation of equations can help students visually see the difference and support algebraic understanding.

Instruction should include providing students with quadratic equations in standard form, vertex form, or intercept form, and have them choose a method for solving (factoring, solving by taking the square root, quadratic formula, completing the square, or graphing) then justify why their solution(s) are real or complex. Students should be exposed to various types of equations and asked to justify how they could determine the type of roots algebraically, as well as, graphically. Reinforcing the definition of a complex number as described in standard M2.N.CN.A.1 can help focus students on what to look for as they solve problems algebraically. Additional support can come from providing students with equations in multiple forms and have them to construct tables representing those equations. They could then inspect those tables and find the zero(s) or justify approximately where the zero(s) would be located. Discussion about the patterns seen in the table of values should help students realize that even though they may not find the exact point on the table which represents the zero, they should be able to determine an approximate location on the table due to the shape of a parabola. It is important to include graphing equations from tables (by hand or using technology) to determine the number and placement of zeros. Equations that are provided should include options of two real roots, one real root, or two complex roots. This process should help students compare and contrast functions written in different forms, and as zeros are revealed from the quadratic equations, students should explain why roots are real or complex.

Level 4:

As students develop a deep understanding of solving quadratic equations they should be provided the opportunity to extend their learning to master solving an equation in a way that is most efficient. Discussion should challenge students to analyze and justify solution methods of equations in different forms based on the ability to quickly recognize the type of roots.

Additionally, instruction should include providing students with real-world quadratic problems in order to apply their problem solving techniques. These real-world problems should include those which produce both real and complex solutions. Students determine if all real solutions are viable in the context of their problem or justify why the function in context would not have a real solution. Explaining their reasoning will help students deepen their understanding of quadratic functions embedded in real-world problems.

STRUCTURE in EXPRESSIONS (A.SSE)

Standard M2.A.SSE.A.1 (Major Work of the Grade)
Interpret expressions that represent a quantity in terms of its context.
M2.A.SSE.A.1a Interpret complicated expressions by viewing one or more of their parts as a single entity.

Scope and Clarifications: (Modeling Standard)
For example, interpret P(1 + r) n as the product of P and a factor not depending on P.
Tasks are limited to quadratic expressions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the parts of a complicated expression as single entities. Identify factor, coefficient, and term.	Recognize arithmetic operations in an expression in order to see the structure of the expression. Describe the relationship between single entities of an expression and the expression.	Interpret parts of an expression (i.e. term, factor, coefficient) embedded in a real-world situation and explain each part in terms of the context. Interpret parts of an expression (i.e. term, factor, and coefficient) and explain each part in terms of the function the expression defines. Explain the structure of an expression and how each term is related to the other terms by interpreting the arithmetic meaning of each term in the expression and recognizing when combining like terms is appropriate.	Interpret expressions in a variety of forms and explain the relationship between the terms and the structure of the expression. Interpret parts of complex expressions with varying combinations of arithmetic operations and exponents by viewing one or more of their parts as a single entity. Write and interpret expressions that represent a real-world context and use the expressions to solve contextual problems. Write expressions in a wide variety of formats and then for each describe the effects each term has considering them first individually

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
			and then considering them as a part of the expression.

Instructional Focus Statements

Level 3:

Seeing structure in expressions is the connecting bridge between arithmetic operations in grades K-8 and algebraic thinking in high school. Instruction should build on students understanding of the relationship between arithmetic operations in expressions and equations. Students should explore a variety of expressions in equivalent forms to see and evaluate the structure in each form. Students should be exposed to exponents of varying degree. This allows them to recognize the attributes of a term in order to combine it appropriately with other like terms. Instruction should expose students to a variety of multiple representations and require students to interpret and explain the relationship between the representations. Students should be challenged with complex, multi-variable expressions to interpret.

Furthermore, students must be able to explain individual terms and interpret that term as a single entity and as a whole expression. Instruction should focus on using the structure of the expression to uncover the attributes of the function it defines. Students should also be able to use precise language to explain the relationship between a verbal description and an algebraic representation. Particular focus needs to be placed on translating words into mathematical expressions and vice versa.

This standard appears in both integrated mathematics I and integrated mathematics II. Tasks are limited to linear and exponential in integrated mathematics I. In integrated mathematics II, students will extend this understanding with quadratics. In future courses, students will experience this standard with radical and trigonometric expressions, so solidifying students' comprehension of the structure of expressions and interpreting the meaning of terms as single entities is imperative.

Level 4:

Students need to be presented with complex expressions that include a combination of different arithmetic operations and interpret in terms of a real-world context. The pinnacle of level 4 understanding is being able understand, interpret, and explain the relationship between equivalent representations of an expression. Students should be able to explain not only the expression in terms of a contextual situation, but also how each term within the expression connects back to the contextual situation.

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Additionally, instruction should focus on relating expressions to real world contexts. For example, students should be given problems that describe contextual situations from multiple perspectives. Students should interpret the contextual situation for each individual perspective and write an expression that represents the context for each. Students should be challenged to interpret the meaning of the expressions created and use them to predict outcomes and solve problems. Instruction should expose students to multiple representations of the expressions by making connections between the equivalent expressions, which will in turn help students recognize the most useful form of an expression depending on context. Students should be challenged to justify why other formats are equivalent and which format is most relevant given the context of the problem.

Standard M2.A.SSE.A.2 (Major Work of the Grade)
Use the structure of an expression to identify ways to rewrite it.

Scope and Clarification:
For example, recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier to evaluate form $(53 + 47)(53 - 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$.
Tasks are limited to numerical expressions and polynomial expressions in one variable.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a numerical or polynomial expression that is equivalent to a given expression.	Rewrite numerical and polynomial expressions into a given form.	Rewrite numerical and polynomial expressions in a different form and explain why rewriting the expression in that form is beneficial.	Generate multiple forms of a single numerical or polynomial expression and explain in both verbal and written form the mathematical reasoning that was employed to rewrite the expression. Additionally, explain which form is most useful and provide mathematical justification.

Instructional Focus Statements

Level 3:

In grade 7, students developed an understanding of using properties to generate equivalent linear expressions. Additionally, they extended this understanding to identify when certain forms of an expression are more useful than others.

In Integrated Math II, students should continue developing the idea that there are often multiple ways to write expressions. Students need to be able to see complicated expressions as built from simpler ones.

Students should be able to provide a mathematical justification for when different forms of expressions are more beneficial. Particular focus needs to be placed on quadratic polynomial expressions as they are a focus of this course.

Much of the ability to see and use structure in transforming expressions comes from learning to fluently recognize certain fundamental algebraic situations.

Developing procedural fluency with quadratics will serve as a cornerstone for future course work and will extend into future courses work with higher powered polynomials.

Level 4:

Students need to be challenged to write polynomial and numerical expressions in multiple forms where the initial expressions increase in difficulty over time. The hallmark of this standard is students being able to communicate the importance and benefit gained from writing expressions in various forms. Students should be able to express what the individual terms within the expression mean and how they relate to terms in the other various representations of the same expression.

Standard M2.A.SSE.B.3 (Major Work of the Grade)
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
M2.A.SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.
M2.A.SSE.B.3b Complete the square in a quadratic expression in the form $Ax^2 + Bx + C$ to reveal the maximum of the function it defines.

Scope and Clarifications: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the correct factorization when given a quadratic expression where A = 1. Determine if the quadratic expression in the form $Ax^2 + Bx + C$ will have a maximum or a minimum.	Choose the zeros of the function it defines, given the factorization of a quadratic expression. Choose the maximum or minimum value of a function defined by a quadratic expression in vertex form $(x-p)^2 = q$.	Factor a quadratic expression to reveal the zeros of the function it defines. Identify equivalent forms of quadratic expressions. Determine the maximum or minimum value of a function defined by a quadratic expression in the form $Ax^2 + Bx + C$ by completing the square.	Explain the most efficient method for factoring a given quadratic expression and justify the reasoning. Recognize algebraically and graphically when a quadratic has 0, 1, or 2 x-intercepts. Determine the maximum or minimum value of a function defined by a quadratic expression by completing the square embedded in a real-world situation and explain the x and y-coordinates of the maximum or minimum within the context of the real-world situation.

Instructional Focus Statements

Level 3:

Students should be able to factor quadratic expressions using various methods. Student understanding of factoring should draw on connecting how and why a quadratic expression and a factored form of the same expression are equivalent. It is important that students gain more than a rote procedural understanding of the process of factoring across all standards involving factoring in Integrated Math II. They need to conceptually understand how factoring works and why factoring works. That said, the primary focus of this standard is connecting the factored form graphically to the zeros of the function the quadratic expression represents. Students are developing an understanding of when and why the factored form is helpful. It is imperative that they connect the terms “zeros” and “x-intercepts” as one is typically geared towards expressions and equations and the other towards graphs. Emphasis should be given to using the factored form of an expression as a means to identify the zeros of the function and then tying that to a graphical representation.

In conjunction with developing proficiency in factoring quadratic expressions, student should be developing an understanding of the value of completing the square. Students need to develop an understanding of when each technique is beneficial along with an understanding of why each is beneficial. Students should realize that completing the square is a method for producing an equivalent form of a quadratic expression which reveals the maximum or minimum of the function defined by the expression. Once again it is important that students connect the quadratic expression and the graph of the function defined by the expression. It is important that there be strong instructional connection with this standard and standard M2.F.IF.B.4. Focus at this level is on understanding the rationale for completing the square as a viable method for solving quadratics as called out in standard M2.A.REI.B.2.

As this is a modeling standard, students should be working with quadratic expressions embedded in real-world situations and let the context of the situation drive the structure of how the expression is written in order to answer a question embedded in the situation.

Level 4:

As students solidify their understanding and become more proficient with working with quadratics embedded in real-world situations, they should be able to not only identify if they need to factor a quadratic or complete the square with a quadratic, but also provide a justification based on the context of the problem to explain why they chose a specific form for the quadratic expression. Additionally, beyond simply finding the zeros of an expression and determining their meaning within a real-world context, students should develop an understanding of how many zeros are generated from quadratic expressions and what the number of zeros can mean in the context of a real-world situation.

As students deepen their understanding of maximum and minimum values for a quadratic embedded in a real-world situation, they should be able to explain the benefit of using completing the square as a method. Students at this level should also possess an understanding of the maximum or minimum values within context and be able to explain their relationship. Ultimately, students should be able to work with real-world situations that ask them to use the same expression and find and interpret intercepts, find and interpreting maximum or minimum values, interpret all values in terms of the context of the problem, and justify how the structure of the expression was used in order for the values to be determined in the first place.

ARITHMETIC with POLYNOMIALS and RATIONAL EXPRESSIONS (A.APR)

Standard M2.A.APR.A.1 (Major Work of the Grade)
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract linear expressions with rational coefficients.	Add two polynomial expressions where both expressions are comprised of the same degree monomials. (i.e. $(x^2+x+3) + (2x^2-3x-4)$) Multiply two binomial expressions. Subtract two polynomial expressions where both expressions are comprised of the same degree monomials. (i.e. $(x^2+x+3) - (2x^2-3x-4)$)	Add polynomial expressions. Subtract two polynomial expressions. Multiply polynomial expressions. Explain what it means for polynomials to be closed under the operations of addition, subtraction, and multiplication.	Add, subtract, and multiply multiple polynomial expressions including situations involving more than one operation. Explain the similarities that exist between adding, subtracting, and multiplying integers and adding, subtracting, and multiplying polynomials.

Instructional Focus Statements

Level 3:

The development of an understanding of polynomials in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions like 7 + 4 and 11 as referring to different entities: 7 + 4 might be seen as a calculation and 11 as its answer. They come to understand that different expressions are different names for the same numbers and that properties of operations allow numbers to be written in different but equivalent forms and that there are often benefits from writing numbers in various forms. They come to see numbers as forming a unified system, the number system.

A similar evolution takes place in algebra. At first algebraic expressions are simply numbers in which one or more letters are used to stand for a number which is unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions as naming some underlying thing. Additionally, they reach the understanding that polynomials form a system in which they can be added, subtracted, and multiplied and that that system is closed for these operations. It is important that students understand this system beyond simply a rote set of steps which allows students to add, subtract, and multiply polynomials.

Level 4:

Students with a deep understanding of polynomials should be able to simplify complex expressions that involve multiple operations. Additionally, they will be able to explain in both verbal and written form the similarities that exist between calculating with integers and adding, subtracting, and multiplying polynomials.

CREATING EQUATIONS* (A.CED)

Standard M2.A.CED.A.1 (Major Work of the Grade)
Create equations and inequalities in one variable and use them to solve problems.

Scope and Clarification: (Modeling Standard)
Include equations arising from linear and quadratic functions and rational and exponential functions. Tasks have a real-world context.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear equation in one variable that represents a simple, real-world situation. Create and solve a one variable linear equation that represents a simple, real-world situation. Create and solve a one variable linear inequality that represents a simple, real-world situation. Solve a simple one variable exponential equation. Identify if a real-world situation can be represented by a linear, quadratic, rational or exponential equation. Determine if the solution to a real-world situation requires a one-	Solve a simple one variable quadratic equation. Solve a simple one variable quadratic inequality. Solve a simple one variable exponential inequality. Solve a simple one variable rational equation. Solve a simple one variable rational inequality. Choose a quadratic or exponential equation to represent a simple, real-world situation. Choose a quadratic or exponential inequality to represent a simple, real-world situation.	Create and solve a one variable linear, quadratic, rational, or exponential equation that represents a real-world situation. Create and solve a one-variable linear or exponential inequality that represents a real-world situation. Create and solve a one-variable quadratic or rational inequality that represents a simple real-world situation.	Create a real-world situational problem to represent a given linear, quadratic, rational, or exponential equation or inequality. Create and solve a one-variable quadratic or rational inequality that represents a real-world situation.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
variable or two variable equation or inequality.			

Instructional Focus Statements

Level 3:

In Integrated Math II, the variety of function types that students encounter allows students to create even more complex equations and work within more complex situations than what has been previously experienced.

As this is a modeling standard, students need to encounter equations and inequalities that evolve from real-world situations. Students should be formulating equations and inequalities, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems. Real-world situations should elicit equations and inequalities from situations which are linear, quadratic, exponential, and rational in nature. As linear and simple exponential functions are a focus in Integrated Math I, it is imperative that students have the opportunity to work with quadratics, rational, and complex exponential equations and inequalities in Integrated Math II.

Level 4:

When given an equation or inequality, students can generate a real-world situation that could be solved by a provided equation or inequality demonstrating a deep understanding of the interplay that exists between the situation and the equation or inequality used to solve the problem.

Additionally, students should continue to encounter real-world problems that are increasingly more complex. Students should be using the modeling cycle to solve real-world problems.

Standard M2.A.CED.A.2 (Major Work of the Grade)
Create equations in two or more variables to represent relationships between quantities; graph equations with two variables on coordinate axes with labels and scales.

Scope and Clarifications: (Modeling Standard)

i. Tasks are limited to quadratic, square root, cube root, and piecewise functions.
ii. Tasks have a real-world context.
iii. Tasks have the hallmarks of modeling as a mathematical practice(less defined tasks, more of the modeling cycle, etc.).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine if the solution to a real-world or mathematical situation requires a one-variable or two-variable equation.	Choose a quadratic, square root, cubed root, or piecewise equation to represent a simple mathematical or real-world situation. Choose a quadratic, square root, cubed root, or piecewise graph to represent a simple mathematical or real-world situation.	Create and graph a two variable quadratic, square root, cubed root, or piecewise equation that represents a mathematical or real-world situation.	Create a real-world situational problem to represent a given quadratic, square root, cubed root, or piecewise equation. Create and graph a two variable quadratic, square root, cubed root, or piecewise equation that represent complex real-world situations.

Instructional Focus Statements

Level 3:

In Integrated Math II, the variety of function types that students encounter allows students to create more complex equations and work within more complex situations than what has been previously experienced.

As this is a modeling standard, students need to encounter equations that evolve from both mathematical and real-world situations. Students should be formulating equations, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to mathematical and real-world problems.

Mathematical situations should elicit equations from situations which are quadratic, square root, cubed root, or piecewise in nature. It is imperative that students have the opportunity to work with each of these function types equally.

Level 4:

One of the most natural situations for students to create an equation or graph from is a real-world situation. Students need to be exposed to variety of real world situations that illicit the wide variety of function types embedded within the Integrated Math II course. Students should encounter real-world problems that are increasingly more complex over time. They should be using the modeling cycle in order to develop and provide justification for their solutions.

Additionally, students should be posed with an equation and then asked to generate a real-world situation that could be solved by a provided equation. Students with this capability are demonstrating a deep understanding of the interplay that exists between the situation and the equation used to solve the problem.

Standard M2.A.CED.A.3 (Major Work of the Grade)
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Scope and Clarifications: (Modeling Standard)

i. Tasks are limited to quadratic, square root, and cube root functions.
ii. Tasks have a real-world context.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose equivalent forms of a given linear real-world formula.	Choose equivalent forms of a given quadratic real-world formula. Choose equivalent forms of a given square root real-world formula. Choose equivalent forms of a given cube root real-world formula.	Rearrange real-world quadratic formulas to highlight a quantity of interest. Rearrange real-world square root formulas to highlight a quantity of interest. Rearrange real-world cube root formulas to highlight a quantity of interest.	Rearrange real-world quadratic, square root, and cube root formulas and explain the benefit of solving the formula for the various variables.

Instructional Focus Statements

Level 3:

In previous grades and courses, students have focused on rearranging linear formulas to highlight a quantity of interest. In Integrated Math II, students should be working with quadratic square root, and cube root equations. As this is a modeling standard, student should be encountering formulas that come from real-world situations. Additionally, students need to be deepening their conceptual understanding of why they might need to write formulas in different ways and what the benefit would be to these various representations of the same real-world formula.

Level 4:

Students need to be exposed to a wide variety of real-world formulas increasing in complexity over time. Additionally, it is imperative that they are able to explain why formulas might need to be expressed in different ways and the benefit that each form provides.

REASONING with EQUATIONS and INEQUALITIES (A.REI)

Standard M2.A.REI.A.1 (Major Work of the Grade)
Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Scope and Clarifications:
Tasks are limited to linear, quadratic, exponential equations with integer exponents, square root, and cube root functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the inverse operations used in solving the equation, given a linear, quadratic, exponential, square root, and cube root equation and a list of steps for the solution method. Choose a possible next step to solve the equation, given a linear, quadratic, exponential, square root, and cube root equation and a partial list of steps for the solution method. Arrange them in the order they should be applied, given a list of steps and their reasons for solving a linear, quadratic, exponential, square root, and cube root equation.	Explain the reasoning for each step, given a quadratic equation and a list of steps for the solution method. Explain the reasoning for each step, given a square root equation and a list of steps for the solution method. Explain the reasoning for each step, given a cube root equation and a list of steps for the solution method. Identify when it can be determined if no solution or infinitely many solutions exists, given an equation and a list of steps for the solution method.	Solve linear, quadratic, exponential, square root, and cube root equations using multiple solution strategies and explain each step in the solution method. Construct a viable argument to justify a chosen solution method for a linear, quadratic, exponential, square root and cube root equation. Compare the steps in each and determine which solution method is most efficient, given an equation with multiple solution methods. Explain why an equation has no solution or infinitely many solutions.	Solve the problem, explain each step in the solution path, and justify the solution path chosen, given a real-world problem and an equation that represents the contextual situation. Compare and contrast two given solution paths to a contextual problem and construct a viable argument on which method is most efficient. Correct the mistakes in the solution path and provide an explanation of the misconception using precise mathematical vocabulary, given a list of steps and an inaccurate solution for a linear, quadratic, exponential, square root, and cube root equation.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain the reasoning for each step, given a linear equation and a list of steps for the solution method. Explain the reasoning for each step when given an exponential equation and a list of steps for the solution method.			

Instructional Focus Statements

Level 3:

In Integrated math II, students should develop a conceptual understanding of solving equations as a reasoning process to determine a solution that satisfies the equation rather than a procedural list of steps. Instruction should focus on students creating and determining solution paths or each unique equation and providing a viable argument to justify the chosen solution path. Students should also be able to explain how, when, and why equations have no solution or infinitely many solutions. To help give meaning to these solution types, discussion should focus on the solution being a value of the variable that makes the equation true. This will help students make the connections that an equation has no solution because there is no value that can maintain equivalency and an equation has infinitely many solutions because all values used for the variable create a true equivalency statement.

Students should understand that a problem can have multiple entry points and instruction should be focused on solving equations using a reasoning process of centered around inverse operations and order of operations. Students develop a conceptual understanding of operations in previous grades and they should deepen their understanding of the interplay that exist between the operations. To illustrate maintaining equivalency, a visual and/or concrete model of a balance scale can be used to aid students in understanding that the same inverse operations are being applied to the whole left side and the whole right side of an equation. Emphasizing equivalency is vital in developing a conceptual understanding of solving equations and preventing common misconceptions. A common misconception is applying an exponent to each term individually instead of applying the power to the entire side of an equation as a quantity. For example, when solving $\sqrt{x-1} = x+2$, students may make the common misconception of raising each individual term to the second power instead of raising the quantity to the second power resulting in a the inaccurate next step of $x-1 = x^2+4$. It is imperative in solving equations for students to understand that inverse operations should be applied to the left side of the equation as one quantity and the right side of the equation as one quantity, not to each term individually. Classroom discussion should address the importance of using grouping symbols when necessary and applying the properties of exponents appropriately as students use inverse operations to solve equations.

Students should understand that the solution path they choose to solve any equation must result in a series of equivalent equations all of which have the same solution set. As students apply inverse operations to solve equations, they should be able to explain why equality holds true when performing the selected operation to both sides of the equation. In this course students should be exposed to linear, quadratic, and absolute value equations, as other function types will be explored in Integrated Math III.

Level 4:

As students develop a deeper understanding of solving equations and explaining their solution methods, instruction can be integrated with the application in contextual situation. Students should be able to construct equations that represent a contextual situation as well as create contextual situations to represent a given equation. As students develop a deep understanding of the relationship that exists between the type of function and the context, they can be given functions embedded in real-world situations. When they are given a contextual situation and an equation, students should be able to determine what each part of the equation represents as it relates to the context. They should also be able to solve the equation and create a viable argument to justify their solution path. Students should understand that there are various ways to solve problems and justifying their steps will help them solidify their understanding of solving equations as well as the most efficient solution path. This standards pairs nicely with standard M2.A.CED.A.1, as it supports the idea of making connections between an equation and its context.

To challenge students to follow a thought process other than their own, they can be asked to critique or correct the solution paths of others. Students will develop a deeper level of understanding if they are given solution paths with incorrect steps in the process or invalid justifications and asked to correct the process or write justifications and defend them.

Standard M2.A.REI.B.2 (Major Work of the Grade)
Solve quadratic equations and inequalities in one variable.
M2.A.REI.B.2a Use the method of completing the square to rewrite any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
M2.A.REI.B.2b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the correct factorization when given a quadratic expression. Choose the correct factorization, given a perfect square trinomial expression. Choose the solution(s) to a simple one variable quadratic equation. Determine if a given value is a solution for a quadratic inequality in one variable.	For an expression written in the form of $ax^2 + bx + c$, where a = 1, find the value of c that makes the expression a perfect square trinomial and rewrite the expression as the square of the binomial. Solve a quadratic equation in standard form using the quadratic formula. Solve a simple one-variable quadratic equation. Choose an interval that represents a solution for a quadratic inequality in one variable.	Use the method of completing the square to rewrite a quadratic equation when $a = 1$, in the form of $(x - p)^2 = q$. Use the method of completing the square to rewrite a quadratic equation when $a \neq 1$, in the form of $(x - p)^2 = q$. Derive the quadratic formula from standard form using the method of completing the square and explain the steps. Solve quadratic equations in one variable using multiple strategies.	Develop a logical argument justifying why completing the square would be the most efficient way to solve a quadratic equation. Complete solution path and justify the steps, when given a problem with partial list of steps for the solution method. Solve one-variable quadratic equations, identify the strategy chosen, and explain why the chosen strategy best suits the initial form of quadratic equation. Solve a quadratic inequality in one variable and explain why the solution set consists of a range of

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Determine if a quadratic equation in one-variable has real solutions or complex solutions write them as $a \pm bi$ for real numbers a and b. Solve a simple factorable quadratic inequality in one variable.	solutions rather than a finite solution.

Instructional Focus Statements

Level 3:

This standards builds on the foundational understanding of the structure of quadratic equations. In M2.A.SSE.B.3, students develop a concrete understanding of using the completing the square process to produce equivalent forms of a quadratic expression. More detailed instructional guidance for completing the square can be found within the M2.A.SSE.B.3 instructional focus document. This guidance hinges on students developing an understanding of the completing the square process through direct modeling strategies such as using models and algebra tiles, including the progression of learning to more abstract strategies such as using algorithms. For M2.A.REI.B.2, students should develop an understanding of how to rewrite quadratic equations so that they are written in standard form into the form $(x - p)^2 = q$ by completing the square. Instruction should focus on rewriting the equation for the purpose of deriving the quadratic formula. In addition, connections should be made between completing the square and vertex form of a quadratic function to support M2.F.IF.B.4 when students graph functions and identify the maxima or minima. To build conceptual understanding, students should be able to explain the steps required to rewrite an equation in vertex form and derive the quadratic formula through the process of completing the square.

It is important that, just like for linear equations, students begin with the understanding developed in M1.A.REI.A.1 that solving any equation is a process. At this level, students should be solidifying their understanding of multiple strategies that can be used to solve quadratic equations. Additionally, they should be providing justification for the strategies (i.e.; by inspection, taking square roots, completing the square, knowing and applying the quadratic formula, and factoring) that they think work best in any given situation. Multiple strategies should not live in isolation but as a part of a much larger conversation around the merits of each. Students should be provided with problems that have been intentionally planned to elicit each of the strategies that can used to solve quadratic equations.

Instruction should also address solving simple factorable quadratic inequalities. This understanding should build on students' prior knowledge of solving quadratic equations and the understanding that inequalities have solution sets that yield a range of solutions rather than a finite solution. Students should be able to determine a solution set(s) that satisfies the inequality and explain what these solutions mean with respect to the inequality.

Level 4:

Students should solidify their understanding of applying multiple strategies to solve quadratic equations and provide mathematical justification to the solution methods they choose. Students should be able to articulate the mathematical underpinnings of the various strategies used in solving quadratic equations. As students focus on justifying and explain their reasoning they should develop a solid understanding of why each process is used.

As students solidify their understanding of solving simple factorable quadratic inequalities, they should also be able to solve more challenging problems that are not factorable using multiple strategies. Students should also be able to explain what the solution set(s) means with respect to the inequality using precise mathematical vocabulary.

Additionally, students should be able to make connection between the various strategies in solving quadratic equations and inequalities. For example, students should be able to explain the similarities and differences in using the quadratic formula and the method of completing the square to solve quadratic equations.

Standard M2.A.REI.C.3 (Supporting Content)

Write and solve a system of linear equations in context.

Scope and Clarifications:
When solving algebraically, tasks are limited to systems of at most three equations and three variables. With graphic solutions systems are limited to only two variables.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the solution to a system of equations in two variables from a graph. Use substitution to determine if a given solution is correct for the system of equations.	Graph two linear equations and find the solution. Solve a system of equations in two variables algebraically through substitution. Solve a system of equations in two variables algebraically through elimination. Identify the solution to a system of equations in three variables from a table.	Write a system of equations in two variables, from a real world situation. Interpret the solution of a system of equations in two variables in context. Solve a system of equations in three variables algebraically. Justify why a system of three linear equations may have one solution, no solutions, or infinitely many solutions. Use technology to solve a system of three linear equations. Write a system of equations in three variables, from a real world situation.	Create a real world scenario to represent a system of equations in three variables. Determine if the solution to a system of equations is reasonable for a given context. Interpret the differences between one solution, no solutions, or infinitely many solutions, given the graph of a system of equations in three variables.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Interpret the solution to a system of equations in three variables in context.	

Instructional Focus Statements

Level 3:

In integrated math I, A.REI.B.2, students wrote and solved systems of two linear equations graphically and algebraically. Instruction should continue to use multiple representations including graphs and tables to help students visualize the solution and support their ability to solve algebraically. When solving a system of two equations graphically, students may approximate this intersection with technology.

In integrated math II, students will expand their understanding of systems to work with systems of equations in three variables. Students should have experience with the process of substitution and elimination, so discussion should help lead students to recognize how these algebraic methods could be used to solve a system of equations in three variables. As the discussion focuses on elimination, students should be led to discover that the process should begin by rewriting the linear system in three variables as a pair of two equations. The solving process will have more layers to solve for more variables and will require students to have the ability to recognize the limitation of only solving for one unknown value at a time. Support will likely need to be given to help them recognize how to combine the value of one variable and continued use of the elimination method to solve for the other variables.

The use of technology is also important when solving a system of three equations. Since the calculator cannot be used to graph the system as it is with a two-variable system of equations, there is an opportunity for matrices to be introduced as a tool for solving systems. Instruction would need to focus on helping students arrange the equations so they can be entered into a matrix, emphasizing the structure and importance of the coefficients of like variables arranged together. This would allow students to enter coefficients and constants into a matrix and then use operations of matrices in the graphing calculator to solve the system of equations.

Instruction should include engaging students in real-world problems where they write and solve equations from context. In some problems, students must determine if an approximation of a solution or an exact solution is most appropriate for the problem and be able to interpret the meaning of these solutions in terms of the context. Additionally, students should be expected to differentiate among problems where there is one solution, no solutions, or infinitely many solutions and justify the meaning or reason for these results.

Level 4:

As students build a strong understanding of how to solve a system of equations in three variables, they should be challenged to extend their understanding of these equations in context by being asked to write a real-world scenario to represent a system. They will continue to demonstrate understanding by choosing the most appropriate method for solving a system and justify their reasoning throughout the solution process.

Although instruction may not include graphing a system of equations in three variables, discussion on the x, y, and z planes and how they are related can help students visualize the system. This experience can help students understand the visual representation of one solution, no solutions, or infinitely many solutions.

Standard M2.A.REI.C.4 (Supporting Content)
Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify whether a system consisting of a linear equation and a quadratic equation would have no solution, one solution, or two solutions, given a graph. Identify the approximate solution to a system of equations from a graph. Identify the solution to a system of equations from a table. Use substitution to determine if a given solution is correct for the system of equations.	Graph a quadratic equation given in any form. Solve a quadratic equation algebraically. Graph a quadratic equation and linear equation on the same coordinate plane to approximate the solution.	Solve a system of a linear equation and a quadratic equation graphically using technology. Solve a system of a linear equation and a quadratic equation algebraically using substitution. Justify why a system consisting of a linear equation and a quadratic equation may have no solution, one solution, or two solutions.	Justify the solution to a system consisting of a linear equation and a quadratic equation using multiple representations: algebraically using the equation, graphically, using tables of values. Write and solve a real world scenario to represent the system, given a system of one linear equation and one quadratic equation. Construct an argument to explain if the solution to a system with one linear and one quadratic equation is reasonable within a given context.

Instructional Focus Statements

Level 3:

Solving a system of one linear equation and one quadratic equation is a natural extension of systems of linear equations (A.REI.C.3). Instruction should continue to use multiple representations including graphs and tables to help students visualize the solutions and support their ability to solve algebraically. When solving a system of two equations graphically, students may approximate this intersection with technology. Algebraically, systems of one linear and one quadratic equation can be solved using substitution. Students should have experiences using the substitution method with linear equations in integrated math I, so building on this understand, students should revisit the property of substitution and how to substitute an expression for the value of a variable. As students solve for y in the use of technology to graph the equations and view the table, discussion should lead them to recognize the equivalency when both equations are equal to y and help them recognize how to set them equal to each other and solve for the remaining variable.

Students discovered patterns that caused linear systems to have no solution, one solution, or infinitely many solutions, so to build on this understanding graphs of different systems containing a linear and a quadratic could be introduced and students could be asked to compare the differences in the number of solutions for these systems. These discussions should help students recognize how the different number of solutions exist; no solution because the graphs do not intersect, one solution caused when the line is tangent to the quadratic, or two solutions when the line will intersect the quadratic twice. Although this is one of the first times students have seen systems of equations that can have two solutions, students have experience solving quadratic equations that have the same results for similar reasons. It should be pointed out that solutions are now the intersection points with the line created by the linear equation as opposed to the x-axis. As students are working with these systems, they should be encouraged to find exact solutions without technology. By attending to precision, students will leave irrational solutions in simplified radical form when the situation arises.

Level 4:

Instruction can extend learning by challenging students to justify solutions to a system of one linear equation and one quadratic equation using different representations of functions. When given a linear and quadratic system represented one way, students would be expected to justify their solution in more than one other way. Options would include substitution of the solution, solving algebraically, using a table values, or graphically.

Once students have a solid algebraic understanding of systems of this type, they could be challenged to write a real-world problem representing the system, construct a table which represents both equations, find the solution and graph the system either by hand or using technology. Students would then need to be able to explain the meaning and justify the reasonableness of the solution in context.

INTERPRETING FUNCTIONS (F.IF)

Standard M2.F.IF.A.1 (Major Work of the Grade)
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Scope and Clarifications: (Modeling Standard)
Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

i. Tasks have a real-world context.
ii. Tasks are limited to quadratic, exponential functions with integer exponents, square root, and cube root functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify intercepts, maximums and minimums when provided a graphical representation of the function.	Identify intervals where a given function is increasing, decreasing, positive or negative when provided a graphical representation of the function. Identify all evident intercepts, maximums and minimums when provided a table of values representing an exponential function with domain in the integers. Identify key features of the graph and interpret the meaning of the key features in relationship to the context of the problem, given a graph of an exponential function	Identify all evident key features when provided a table of values representing a quadratic, square root, or cube root function. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a quadratic function embedded in a real-world context. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values	Identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given a quadratic function embedded in a real-world context, graph the function. Identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given a square root function embedded in a real-world context, graph the function. Identify key features of the graph, and interpret the meaning of the

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	with domain in the integers embedded in a real-world context. Identify evident intercepts, maximums and minimums when provided a table of values representing a quadratic, square root, or cube root function.	representing a square root function embedded in a real-world context. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a cube root function embedded in a real-world context. Sketch a graph of the function, given a verbal description of the key features of a quadratic, square root, or cube root function.	key features in relationship to the context of the problem when given a cube root function embedded in a real-world context. Create a real-world context that would generate a function with the provided attributes, given key features of quadratic, square root, or cube function.

Instructional Focus Statements

Level 3:

Functions are often described and understood in terms of their key features and behaviors. Instruction for this standard should in part focus on building on the knowledge gained in Integrated Math I around identifying key features and behaviors from both graphs and tables and extend this understanding to new function families. In Integrated Math I, students focused on linear, absolute value, and exponential functions with domain in the integers. The new part of this standard for students will be in the function families as opposed to the types of key features/behaviors. Thus, it is important to note that the overarching concept of key features and behaviors is not new to students.

As in Integrated Math I, instruction should extend beyond simple identification from isolated graphs and tables. As this is a modeling standard, students need opportunities to develop an understanding of the relationship between key features/behaviors and the real-world situation that the function models. The focus should be on developing a strong understanding of the relationship between key features/behaviors and their meaning within real-world situations. Additionally, instruction should provide students with an opportunity to develop an understanding of not only how to identify key features/behavior in graphs and tables, but also on how to generate a graph when provided the key features/behaviors.

Instruction can be very nicely paired with standard M2.F.IF.B.4 where students generate quadratic, square root, and cube root graphs from real-world situations. This pairing allows students the opportunity to generate a graph from a real-world situation, identify key features/behaviors, and then discuss their meaning as related to the real-world situation.

Level 4:

As students develop a deep understanding of this standard, they should be exposed to increasingly more complex real-world situations. Students should begin to create their own real-world scenarios that generate functions with a pre-determined list of key features/behaviors. Additionally, students with a deep understanding of this standard can interpret key features/behaviors from non-traditional quadratic, square root, and cube root functions embedded in real-world situations.

Standard M2.F.IF.A.2 (Major Work of the Grade)
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★

Scope and Clarifications: (Modeling Standard)
For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Tasks are limited to quadratic, square root, cube root, piecewise, and exponential functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain the difference between domain and range. Identify the domain and range from a table of values. Identify the domain and range from a discrete graph of ordered pairs. Identify the domain and range, given a mapping. Identify the domain and range from a set of ordered pairs. Explain the difference between a continuous function and a discrete function.	Explain how domain relates to the definition of a function. Identify the domain, given the continuous graph of the function. Explain why a function is continuous or discrete given its graph.	Explain how the domain relates to the graph of a function. Explain why a function is continuous or discrete, given an equation. Describe how a function's domain is affected when situated within a context. Explain if a function is continuous or discrete, given a context.	Determine an appropriate domain and range, given a function in context. Create a contextual situation to describe a function with a given domain and range. Using the definition of discrete and continuous, compare and contrast sequences and the functions used to model them.

Instructional Focus Statements

Level 3:

Instruction began on this modeling standard in integrated math 1 when students related the domain of a function to its graph and where applicable, to the quantitative relationship it described. In integrated math 1 it is important that students have a solid understanding of domain and range. They are provided tables, graphs, ordered pairs and mappings to help identify the domain and range of linear, piecewise, and exponential functions. Instruction in integrated math 2 should reinforce these concepts and apply them to a larger variety of functions types including quadratic, square root, cube root, piecewise and exponential functions. As students identify the domain and range, multiple representations will give students a deep understanding and provide a basis for rich classroom discussion. Teachers should have students explain their reasoning as to why they have identified the domain and range in these varied forms. After students have a clear understanding of how to identify the domain and range of these additional functions, they should relate this to the definition of a function. Discussion should include comparing and contrasting discrete and continuous functions and identifying them from a graph and real-world situations.

The following might be example to help students apply this standard. During "March Madness", the NCAA basketball tournament begins with 64 teams. If half of the teams are eliminated each round, how many teams are left at the end of each round? How many rounds will occur before a champion is announced? Students should determine whether the domain is continuous or discrete and graph the function. In addition, careful attention must be paid to real-world problems where the domain might be continuous, but also restricted because of the context. An example might be a ball's height when thrown, h(t), with respect to time. The domain would be continuous, but given the context, t must be greater than or equal to 0. As students become proficient with different function families, discussion should help them realize when and why domain and range do not both have to be continuous or discrete. This is a good place to incorporate step functions and explain how the domain might be is continuous while the range is discrete.

Level 4:

As students are exposed to a variety of real-world problems, they begin to realize how unique and different every problem can be, but that every modeling situation will have a domain and range. To deepen their understanding, students can asked to determine the domain and range of a variety of scenarios within a context. They should also have practice with justifying whether the function is continuous or discrete. As they develop a good understanding of domain and range, opportunities should be provided for students to create their own scenarios when given a domain and range. In addition, students should construct an argument explaining why their scenario represents the given domain and range. Instruction should also focus on connecting arithmetic and geometric sequences with the functions that model them. Students should justify why these sequences are discrete, while linear and exponential functions are continuous.

Examples which might be used to support understanding of this standard would include representing the placement of rivets on the parabolic support of a bridge because this would need to be represented by a discrete function and the domain and range are restricted to positive integer values. A continuous example might be a punter kicking a football over time (x) compared to height (y) because this also restricts the domain and range to positive values. As students become proficient in determining domain and range and graphing functions, they should be able to create their own scenarios with both discrete and continuous domain and range.

Standard M2.F.IF.A.3 (Major Work of the Grade)
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Scope and Clarifications: (Modeling Standard)

i. Tasks have a real-world context.
ii. Tasks may involve quadratic, square root, cube root, piecewise and exponential functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the average rate of change for an exponential function when given a symbolic representation, table, or graph. Choose the estimated rate of change when given a graph of an exponential function.	Calculate the average rate of change of an exponential function when given a graph. Interpret the rate of change for an exponential function in terms of a real-world context. Choose the estimated rate of change for a specific interval when given a quadratic function.	Calculate average rate of change when given an equation or table of a quadratic, square root, cube root, piecewise and exponential functions. Interpret the average rate of change of a quadratic, square root, cube root, piecewise and exponential functions. Estimate the average rate of change for a specific interval of a quadratic, square root, cube root, piecewise and exponential functions.	Identify the average rate of change for specific intervals of a function as being greater or less than other intervals of the same function. Compare the average rate of change of multiple intervals of the same function and make connections to the real-world situation. Create a contextual situation and identify and interpret the average rate of change with a specific interval.

Instructional Focus Statements

Level 3:

In grades 6 and 7, students began developing the understanding of ratios and proportional relationships. Their understanding of rate of change involved both ratios and proportions using similar triangles to show the additive and multiplicative conceptual underpinnings of the concept. In grade 8, students extended this understanding to functions by examining rate of change in linear functions. In high school, students should solidify this understanding for linear functions and generalize this concept to applying to additional function types. Students should make the connection that the rate of change is the ratio of the change between the dependent and independent variable. For linear functions, students have discovered that this ratio of change is constant between any two points on the line. Students should now make the connection that, for non-linear functions, the ratio of change is not constant due to the functions curvature. This results in the ability to calculate the average rate of change over a specified interval. For example, for the quadratic function $f(x) = x^2$, the average rate of change from $x = 1$ to $x = 4$ is $\frac{f(4)-f(1)}{4-1} = \frac{16-1}{4-1} = \frac{15}{3} = 5$. This is the slope of the line from (1,1) to (4,16) on the graph *f*. If *f* is interpreted as the area of a square of side x, then this calculation means that over this interval the area changes, on average, 5 square units for each unit increase in the side length of the square.

It is imperative that students gain a conceptual understanding of the average rate of change for a specified interval for non-linear functions. To grasp this idea, students should draw illustrations of the graph and the secant line connecting the intended endpoints. Students should not only be able to calculate the average rate of change, but they should also be able to generate a visual representation and use the visual representation to estimate the average rate of change over a specified interval. Students will gain a deeper conceptual understanding when they compare their estimations to the actual average rate of change for a non-linear function. As students solidify their understanding, they should be able to explain what the average rate of change means in the context of a problem when given symbolic representations, tables, graphs, or contextual situations. As students use multiple representations to evaluate the average rate of change, they should be able to explain the relationship between the multiple representations using both appropriate mathematical language and appropriate justifications.

Level 4:

Students should extend their understanding of average rate of change by comparing the average rate of change of one interval to another interval of the same function. Students should also further their understanding by creating their own contextual situations and interpreting the average rate of change for a significant interval. Students should be intentional in determining which interval or intervals they select and explain the importance of the interval(s) with respect to the context using both precise mathematical vocabulary and precise justifications.

M2.F.IF.B.4 (Supporting Content)
Graph functions expressed symbolically and show key features of the graph, by hand and using technology.
M2.F.IF.B.4a Graph linear and quadratic functions and show intercepts, maxima, and minima.
M2.F.IF.B.4b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
M2.F.IF.B.4c Graph exponential and logarithmic functions, showing intercepts and end behavior

Scope and Clarifications: (Modeling Standard)
M2.F.IF.B.4a – Tasks are limited to quadratic functions.
M2.F.IF.B.4c – Tasks are limited to exponential functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Use characteristics of the symbolic representation of a function to distinguish function type and behavior of the graph. Recognize the parent function from a graph of a square root, cube root, absolute value, piece-wise or exponential function.	Sketch the graph of a quadratic function given intercepts and extrema. Describe the behavior of the graph of the square root function by explaining the inverse relationship between squaring a number and taking the square root of a number. Describe the behavior of the graph of the cube root function by explaining the inverse relationship between cubing a number and taking the cube root of a number. Identify key features, such as shape, intercepts, extrema, and end behavior, from a graph of a square root and cube root function.	Graph a quadratic, square root, cube root, piecewise-defined, absolute value, and exponential function by hand and using technology. Attend to precision when illustrating intercepts, maxima, minima, and determine the domain, range, and end behavior of the function.	Explain the relationship that exists between a contextual problem and the key features of a graph for a linear, quadratic, square root, cube root, piece-wise-defined, absolute value, and exponential function. Critique graphs drawn by others to ensure key features are shown efficiently and appropriately. Write the corresponding function symbolically, given a graph. Explain restrictions on domain and range in context.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Explain how the absolute value function is an example of a piecewise-defined function. Identify key features, such as extrema and intercepts, from a graph of a piecewise-defined function. Infer restrictions on the domain and range from a graph. Identify key features, such as intercepts, extrema, and end behavior from a graph of a quadratic function. Identify the asymptote, given a graph of an exponential curve. Recognize the parent function from a graph of a transformed quadratic, square root, cube root, absolute value, and exponential function.		

Instructional Focus Statements

Level 3:

In Integrated Math I, students used functions to model the relationship that exists between quantities and construct a function to model a linear relationship. Students also determine if a function is linear or nonlinear, and they have experience interpreting and representing functions algebraically, numerically, graphically, and verbally. Integrated Math I and Integrated Math II, students will be introduced to additional function families and more key features such as extrema and end behavior. Intercepts, shape, domain, and range take on greater meaning to students through the exploration of a variety of functions. Therefore, it is important for instruction to provide tasks that allow students to continue to explore the behavior and varying parameters of a wide variety of functions. Providing students with models in context, such as connecting the quadratic function to an area problem and the cube root function to solving volume problems, will help develop the meaning of key features and identifying them from a graph, a table, or a verbal description as supported by standard M1.F.IF.B.3. To meet the rigor of this standard, students should be given the opportunity to work with functions that vary in their symbolic representation such as standard form, factored form, and vertex form of a quadratic function. This will help students have access to the problem regardless of the symbolic representation, which is further developed in standard M2.F.IF.B.5.a as students identify key features through algebraic manipulation.

Students should be able to graph functions by hand and with the use of technology. It is imperative for the teacher to model how to graph with a graphing calculator or other graphing device. This is the first time students will use technology to graph a function type other than a linear function. Furthermore, ample time must be given for students to explore how a table of value can be helpful in identifying key features, domain, and range from a graph. The use of technology should allow students to explore problems whose key features are irrational values, which can be located with the use of a device.

Integrated Math I students focused on linear parent functions. Instruction in Integrated Math II should build on the concept of parent functions. This will help students make the connection of how transformations affect the graph, equation, and table of a function, which is explored in standard M2.F.BF.B.2. . This standard can be integrated in instruction as students are presented with problem types whose symbolic representation varies and asked to identify the parent function and describe the transformation from its original, non-transformed graph. This will help students attend to precision as they graph functions of many types and use their understanding of transformations to support the reasonableness of their graph. Instruction should provide ample opportunity for students to compare and contrast the graphs of functions, and it should help them efficiently recognize a parent function when expressed symbolically and graph it fluently.

Students may struggle with domain restrictions. Integrating instruction with standard M2.F.IF.A.2 can help support student understanding. Additionally, technology can support student conceptual understanding of domain and range restrictions when students graph multiple functions on the same coordinate plane. For instance, have students graph $y = \sqrt{x}$, $y = \sqrt{x+4}$, and $= \sqrt{x}$ + 4 simultaneously and make observations about the domain and/or range by looking at the graph and analyzing the table. Students should encounter enough examples for them to discover the relationship between the algebraic representation and the resulting domain and range. Examples should be given so students discover the relationship between the algebraic representation and the resulting domain and/or range to help students understand piecewise-defined functions. For piece-wise functions, instruction should begin by defining a simple piecewise function with two constant functions in context connections between the graph and the domain. Students will need to connect their prior knowledge of inequalities when working with domain in piece-wise functions.

Additionally, standard M1.F.LE.A.2 introduces students to exponential functions. Instruction in Integrated Math II should build on students' prior knowledge of geometric sequences in order to graph exponential functions to highlight in key features. Instruction should connect the initial value with the y-intercepts and the end behavior as x approaches negative infinity with the horizontal asymptote.

Level 4:

Students should extend their conceptual understanding of key features of graphs by connecting key features to the relationships that exists in the contextual problems. Using their knowledge of multiple representations, students should be able to provide a graph, table, equation, and verbal representations of a contextual situation. Instruction should include posing purposeful questions asking them to show and describe key features from their created problem in context. Students should be given the opportunity to look at graphs drawn by others so they can analyze and critique their peers work. Through the analysis of many graphs, students should develop an understanding of when key features are efficiently and effectively represented, and, if not, provide a suggestion for representing them more appropriately.

M2.F.IF.B.5 (Supporting Content)
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
M2.F.IF.B.5a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
M2.F.IF.B.5b Know and use the properties of exponents to interpret expressions for exponential functions.

Scope and Clarifications:
For example, identify percent rate of change in functions such as $= 2x$, $y = (1/2)x$, $y = 2 - x$, $y = (1/2) - x$.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Factor quadratic expressions with a leading coefficient of 1. Identify zeros of a quadratic function in factored form using the zero-product property. Identify and factor out the greatest common factor from an expression when appropriate. Identify the y-intercept for a quadratic function written in standard form. Use the properties of exponents to rewrite exponential expressions.	Factor quadratic expressions when the leading coefficient is not 1. Factor quadratic expressions in the form of the difference of two squares. Complete the square to convert a quadratic function from standard form to vertex form when the leading coefficient is 1. Identify the vertex of a quadratic function written in vertex form. Identify an exponential function written in symbolic form as an exponential growth or decay. Identify percent rate of change in an exponential function.	Interpret the meaning of zeros, y-intercept, extreme value, and the axis of symmetry in the context of a real-word problem. Complete the square to convert a quadratic function from standard form to vertex form when the leading coefficient is not 1. Recognize which form of a quadratic function is most appropriate for revealing certain properties. Explain what the leading coefficient represents and the relationship that exists between the standard, factored, and vertex forms of a quadratic function.	Write a quadratic function from a real-world context given zeros or the extreme value, and one additional point. Explain multiple methods to reveal each property from standard, factored, and vertex form without converting to another form. Identify reference points in non-standard forms of exponential functions.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Determine if the context of a real-world problem represents exponential growth or exponential decay.	. Recognize that various forms of a function are equivalent expressions and that each can be represented by the same graph. Rewrite an exponential function to reveal the percent rate of change. Rewrite an exponential function to reveal the y-intercept. Write an exponential function given any point and the percent rate of change.	

Instructional Focus Statements

Level 3:

In previous grades, students convert linear functions to slope-intercept form to reveal two characteristics of linear functions: slope and y-intercept. In Integrated Math II, students look for and make use of structure to rewrite quadratic functions to reveal characteristics about their graphs. Therefore, this standard relies on a deep understanding of standard M2.A.SSE.B.3, in which students choose and produce equivalent forms of quadratic expressions. Teachers should build upon this by supporting students in converting among standard, vertex, and factored forms and understanding which form is most appropriate for revealing certain key features.

As students learn to factor quadratics, they should also understand how applying the zero-product property to factored form aids in finding the zeros efficiently. This concept can be revealed to students by having them graph a quadratic and its linear factors on the same coordinate plane to visually see that the linear factors share the same zeros as the quadratic. Zeros are the input value(s) that produce an output value of 0, which could represent the break-even point of a profit function, the time when a projectile hits the ground, or a length of a rectangle that creates an area of 0.

By completing the square, students can write quadratic functions in vertex form to reveal the vertex and axis of symmetry of the function. Visually, students see that the vertex is the maximum or minimum of the function, which is important in a variety of contexts. For example, it can reveal the unit price that maximizes profit, the time when a projectile reaches its maximum height, or the length of a rectangle that maximizes the area. Teachers can also help students understand vertex form by making connections to transformations of functions and why a function $f(x-2)-4$ shifts the function right 2 and down 4, which will aids in understanding why the vertex of $y = (x-2)^2 - 4$ is (2, -4). Thus, teachers can pair instruction with concepts of transformations learned in standard M2.F.BF.B.2.

Instruction should also focus students' attention on the leading coefficient and how it appears in all three forms. Therefore, the concavity of a quadratic is revealed in each. Students should interpret the concavity as determining the shape of the quadratic and whether it has a maximum or minimum. Teachers should provide a variety of contexts in which the type of variables differ so that students can develop a deep understanding of how each component relates to the real-world context. Moreover, matching functions given in various forms with possible graphs that they could represent is a useful task to help students visualize how each form of a quadratic reveals key features of the function.

In Integrated Math I, students use the properties of exponents to rewrite exponential expressions, interpret key features of an exponential graph, and interpret the parameters in an exponential function in terms of a context. Students build upon this prior knowledge and combine these skills to rewrite exponential functions to reveal key features of the graph and interpret them in a context. Students should be able to identify the y-intercept and percent rate of change (i.e., percent increase or decrease) by rewriting an exponential function into the form $y = ab^x$. Teachers should provide students with opportunities to use exponent properties to convert exponential functions into the form $y = ab^x$. For example, given the exponential function $y = (1/3)^{-x}$, students should write this as $y = ((1/3)^{-1})^x$ or $y = 3^x$. Therefore, students should identify the percent rate of change as a 200% increase. In other cases, students should identify the percent rate of change in a context. An example might be to determine the percent rate of change in the population of a small town, when the population is represented by $p(t) = 15000(.978)^t$. A common misconception held by students is that it represents an increase of 97.8% rather than a decrease of 2.2%. Teachers can model how to write the function as $y = 15000(1-.022)^t$ to help students see why it is a decrease of 2.2%.

Student can also use exponent properties to reveal the y-intercept of an exponential function. For instance, given the function $y = 2(3)^{x+1}$ teacher should help students recognize that the function can be rewritten as $y = 2(3)^1 * (3)^x$, which is equivalent to $y = 6(3)^x$. Thus, the y-intercept is (0,6). One useful strategy to support students in separating an exponential expression into the product of two exponential expressions is to first provide them with examples of combining the product of two expressions. By doing so, student can use the exponent properties in ways they are accustomed to prior to reversing the thought process. Although separating the expressions can be useful, teachers should remind students that another strategy to find the y-intercept is to substitute 0 in for x, which is generalizable to all functions.

These concepts builds upon standard M1.F.BF.A.2, in which students write geometric sequences with an explicit formula and use them to model situations. Therefore, if students are given the ratio and a term in the sequence, they can use this concept to build the function without finding the first term. For example, given the 7th term is 30 and the common ratio is 2, students should be able to write the function $y = 30(2)^{x-7}$ because an input of 7 will result in an exponent of 0, which will produce an output of 30. This can help students work efficiently by making use of the structure of an exponential function.

<u>**Level 4:**</u>

Students with a deep understanding of the three forms of a quadratic should be able to create quadratic functions when given enough information. For example, students could be given one zero and the vertex. Using the symmetry of the graph, teachers can focus students' attention on finding an additional zero to build factored form. From there, they can substitute in the x and y value of the vertex to find the leading coefficient, a. Teachers should create functions within a context so that students continue to see the key features of quadratic functions within various contexts.

Although converting among the three forms is an important skill, students at this level should be able to locate the critical features of a quadratic graph directly from each of the three forms. For example, instruction should provide the opportunity for students to explore how to find the vertex of a quadratic written in factored form by averaging the two zeros in order to find x and then substituting the x-value into the function to find the corresponding y-value. This is also a great opportunity for teachers to remind students that the quadratic formula can be used to find the zeros directly from standard form.

As students develop a deep understanding of exponential functions, they should recognize reference points in non-standard forms of exponential functions. For example, given the function $y = 3(2)^{x-5}$, students should recognize that the function contains the point (5,3). There are multiple ways that teachers can help students understand this principle. First, students can see the function $y = 3(2)^{x-5}$ as a horizontal translation of the function $y = 3(2)^{x}$, which has a y-intercept of (0,3). Therefore, instead of having the point (0,3), the function would contain the point (5,3). Second, students should know that anything (except 0) raised to the 0 power is 1 and use this fact to see how making the exponent 0 simplifies the entire second factor to 1. Thus, by inspection, students should quickly see that an input of 5 produces an output of 3 and explain how this concept can be generalized to other exponential functions.

Standard M2.F.IF.B.6 (Supporting Content)
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Scope and Clarifications:
i) Tasks do not have a real-world context.
ii) Tasks may involve quadratic, square root, cube root, piecewise, and exponential functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the y-intercept of a function from multiple representations. Identify the slope of a linear function from multiple representations. Identify the asymptote of an exponential function from multiple representations. Describe connections among multiple representations of a linear function. Compare properties of two linear functions each represented in a different way.	Identify the zeros of a function from multiple representations. Identify the vertex of a quadratic function from multiple representations. Identify the percent rate of change of an exponential function from multiple representations. Describe connections among multiple representations of a quadratic function. Describe connections among multiple representations of an exponential function. Describe connections among multiple representations of a piecewise-defined function.	Compare properties of two exponential functions each represented in a different way. Compare properties of two piecewise-defined functions each represented in a different way. Compare properties of two quadratic functions each represented in a different way. Compare properties of two square root functions each represented in a different way. Compare properties of two cube root functions each represented in a different way. Compare properties of two functions from different function	Compare properties of two functions within a context. Use precise mathematical vocabulary to explain the relationships of the various representations of a function.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Move fluently among multiple representations of a function.	families each represented in a different way.	

Instructional Focus Statements

Level 3:

Prior to comparing properties of two functions represented in different ways, students need to first identify properties of functions and make connections between different representations of the same function. This is an important standard with respect to achieving access and equity for all students. Teachers should represent a function in multiple ways, especially for English language learners, learners with special needs, or struggling learners, because math drawings and other visuals allow more students to participate meaningfully in the mathematical discourse in the classroom. As students move fluently between representations they must consider relationships among quantities and how each representation provides a unique perspective of the function. Teachers can foster this way of seeing mathematics by having students discuss the similarities among representations that reveal the key features of a function that persist regardless of the form. Through these discussions students can determine which representations are most appropriate for revealing certain key features of the function.

In grade 8 and Integrated Math I, students compare properties of two linear, piecewise, and exponential functions each represented in a different way. Once students have a strong understanding of the various representations of quadratic, cube root, and square root functions in integrated math II, they can begin to compare properties of two functions represented in different ways. For example, given a graph of one quadratic function and a table of another, a student should be able to compare their y-intercepts. One strategy that can sometimes be useful is to convert one or both to a different form so that both functions are represented the same way. As students begin to grasp this concept, it is important that teachers provide students with examples that include each function type, with some situated within a context. Therefore, comparing properties in different representations further supports students' understanding of each function type, which means this standard can be paired nicely with other standards that focus on properties and graphs of quadratic, cube root, square root, piecewise, and exponential functions, such as standards M2.F.IF.A.1 and M2.F.IF.B.4. As students recognize various function types in multiple representations, discussion should lead to the comparison of functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a quadratic function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties.

Level 4:

Students with a deep understanding of the various function types and representations should also be able to compare functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a quadratic function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties. Once conclusions are formed, teachers can ask further questions related to the context. For example, given a graph of a linear function and an algebraic representation of a piecewise-defined function each describing the cost of a cell phone plan, decide which plan is better. Students should be given the opportunity to describe how to identify function types and compare the properties of functions in various forms. At this level, teachers should expect students to use precise mathematical vocabulary to describe and justify these relationships and qualities.

Building Functions (F.BF)

Standard M2.F.BF.A.1 (Major Work of the Grade)
Write a function that describes a relationship between two quantities

M2.F.BF.A.1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
M2.F.BF.A.1b. Combine standard function types using arithmetic operations.

Scope and Clarifications: (Modeling Standard)
For M2.F.BF.A.1a:
i) Tasks have a real-world context.
ii) Tasks may involve linear and quadratic functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Write a function defined by an expression, a recursive process, or steps for calculation to model a linear relationship, given a real-world context. Identify the independent and dependent variable in a real-world context. Combine like terms of two linear functions to build a new function.	Recognize when a quadratic function should be used to describe the situation, given a real-world context. Combine a linear function and quadratic function using arithmetic operations to build a new function.	Write a function defined by an expression to model a quadratic relationship, given a real-world context. Compare key characteristics of real-world contexts that can described by various types of functions. Combine multiple functions using arithmetic operations to write a function that describes a real-world situation with multiple steps. For example, combine a linear function and an exponential function.	Create a real-world context that would generate the given function, given a function defined by an expression, a recursive process, or steps for calculation. Explain the various ways a function can be defined and in what real-world situations they would be appropriate. Justify why specific types of functions should be used in combination to describe a given real-world context.

Instructional Focus Statements

Level 3:

In Integrated math I, students create linear and exponential functions from a real-world context defined by an explicit expression, recursive process, or steps for calculation. In integrated math II, students should also extend their understanding to quadratic functions as well as combining function types using arithmetic operations. Instruction for this standard should focus on creating functions from a real-world context and recognizing when multiple functions need to be combined to describe multiple steps. Combining multiple functions is not limited to linear and quadratic functions. For example, students should also be able to combine a linear function and an exponential function to build a new function.

If given a table of values describing a single function, students should first recognize which type of function the table of values represents. Teachers should focus students' attention on the relationship between consecutive points to see if there is a common first difference or constant additive change (linear function) or a common second difference (quadratic function). Once students identify the function type, teachers can then help students begin to write the function given the common first difference, second difference, and other information from the table.

To build coherence, it is important that teachers make connections between linear functions and arithmetic sequences. Thus, instruction should build upon M1.F.BF.A.2, where students generate explicit arithmetic formulas to model situations. Both linear functions ($y = ax + b$) and arithmetic sequences ($a_n = a_1 + d(n - 1)$) describe additive changes, and students should make connections between the two. For example, b is equivalent to a_0 and a is equivalent to d. Students should understand the similarities, but instruction should also help students realize an important difference: arithmetic sequences are discrete while linear functions are continuous. This can be done by comparing the graphs of an arithmetic sequence and a linear function. Moreover, combining two linear functions using multiplication creates a quadratic. Teachers should show students that the graphs of the two linear factors ($h(x)$ and $g(x)$) have the same zeros as the quadratic ($f(x)$), which provides a nice visual of the zero product property. In addition, since $h(x) * g(x) = f(x)$ for all values of x, teachers can have students multiply the functions graphically by multiplying the y-values of $h(x)$ and $g(x)$ for any x-value to produce the y-value of $f(x)$. For example, $h(0) * g(0) = f(0)$. Introducing quadratics in this way will help support students in understanding why factoring can be used to find the zeros of a quadratic (M2.A.SSE.B.3).

Level 4:

As students develop a deep understanding of this standard, they should be able to create a real-world scenario given a function or combination of functions. Moreover, they should be able to describe which characteristics of their scenario correspond to each part of the given function. For example, given $y = x^2 + 2x$, a student might create a scenario similar to the following: tiles are laid down to create a pattern. In pattern 1 there is a 1x1 square and 2 tiles next to the square on both sides, making a total of 3. In pattern 2, there is a 2x2 square with 2 sets of 2 on both sides of the square. This pattern continues as larger squares are formed with 2 sets of x tiles on both sides of the square. y represents the total number of tiles in the x^{th} pattern. Students should also recognize that this is the same as $x(x + 2)$, or a rectangle with side lengths of x and $x + 2$. As students continue to work with combining functions, teachers should have students recognize that when combining functions using some operations, the original parent function is maintained, while other operations create new function types. For example, teachers should have students compare adding two quadratic functions with multiplying two quadratic functions.

Standard M2.F.BF.B.2 (Supporting Content)
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Scope and Clarifications:
i) Identifying the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x + k)$ for specific values of k (both positive and negative) is limited to linear, quadratic, and absolute value functions.
ii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear, quadratic, square root, cube root, and exponential functions.
iv) Tasks do not involve recognizing even and odd functions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Describe, using precise mathematical vocabulary, transformations that would map a geometric figure to its image. Write the function defined by $f(x) + k$, given the function and a positive value of k. Write the function defined by $kf(x)$, given the function and a positive value of k. Write the function defined by $f(x + k)$, given the function and a positive value of k.	Compare $f(x)$ and $f(x) + k$ and illustrate an explanation of the effects on the graph using technology. Compare $f(x)$ and $kf(x)$ and illustrate an explanation of the effects on the graph using technology. Compare $f(x)$ and $f(x + k)$ and illustrate an explanation of the effects on the graph using technology	Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $f(x) + k$. Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $kf(x)$. Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $f(x + k)$. Determine the value of k for a specific vertical or horizontal translation or vertical stretch or compression, given two graphs, the image and pre-image.	Write the equation of a function given the graph by identifying the transformation(s) to the parent function. Apply transformations to a function that has already been transformed. Explain why changes to the argument of $f(x)$ affect the input values and changes outside the function affect the output values.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Describe multiple effects on a graph for specific values of a, h, and k given two functions, $f(x)$ and $af(x+h)+k$.	

Instructional Focus Statements

Level 3:

In grade 8, students verify experimentally the properties of rotations, reflections, and translations of simple figures. Students expand on this concept in algebra I by applying transformations to functions and describing the transformations using function notation. Students must have a deep understanding of function notation and how to write a function defined by function notation (e.g., given $f(x)$, write the function $f(x+3)$). Therefore, building upon M1.F.IF.A.2 in integrated math II, students will see how function notation can be used to generalize transformations to all functions.

To understand how a, h, and k impact the graph of $f(x)$ when compared to $af(x+h)+k$, students can use technology (i.e., calculator or online graphing tool) to experiment with $f(x)+k$, $af(x)$, and $f(x+h)$. As students vary one value at a time, they can begin to discern how each component affects the graph of $f(x)$. Careful attention should be made to why h translates the graph $-h$ units horizontally. One explanation is to compare $f(x)$ with $f(x-5)$ and see that $f(3)$, for example, will produce the same output value as $f(8-5)$. In this example, an input of 3 into $f(x)$ is equivalent to an input of 8 into $f(x-5)$, and 8 is 5 units to the right of 3, not left. Students can think about it as having to undo what has been done to x inside the argument, which leads nicely to understanding $f(kx)$ in future courses. Meanwhile, $f(x)$ represents the output values. So, any operations performed to $f(x)$ outside the argument only affect the y values, which results in vertical transformations. Once students understand the effects of each component individually, they should then attempt to describe changes to a graph involving multiple transformations at once.

Additionally, connections between M2.F.BF.B.2 and M2.A.SSE.B.3 should be made. Converting to the vertex form of a quadratic reveals the transformations being made to $y=x^2$, which allows students to easily locate the vertex and determine the concavity of the parabola. Making this connection will help support a deep conceptual understanding of vertex form. Students should realize that the same transformations will be applied to other function types such as polynomials and trigonometric functions in future courses.

Level 4:

Students with a deep understanding of this standard should be able to write the equation of a function given the graph by identifying the transformation(s) to the parent function. Teachers should focus students' attention on the order of each transformation. For example, given $-f(x)+9$, the graph is first reflected across the x-axis, then shifted up 9, rather than shifted up 9 then reflected across the x-axis due to the order of operations. It is also important that teachers place an emphasis on factoring out b from inside the argument so that the horizontal shift can be found. For example, write $(2x-6)^2$ as $(2(x-3))^2$ instead, revealing a horizontal shift of 3 to the right, not 6. Teachers should help students understand that transformations can be made to functions that have already been transformed. For example, given $f(x)=(x+2)^2$, write an equation for $f(x-5)$ and describe the overall change in the graph. In addition, students at this level should be able to explain concepts such as why changes inside the argument of a function have the inverse effect on a graph. Taken collectively, these students should understand a function as a process that generates output values from particular input values. Building on this understanding, students should make connections with which transformations perform operations to the input values prior to the function's operations and which transformations perform operations to the output values after the function has been applied.

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SIMILARITY, RIGHT TRIANGLES, and TRIGONOMETRY (G.SRT)

Standard M2.G.SRT.A.1 (Major Work of the Grade)
Verify informally the properties of dilations given by a center and a scale factor.

Scope and Clarification:
Properties include but are not limited to: a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center of the dilation unchanged; the dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Distinguish between a dilation and a translation, reflection, or rotation. Choose a dilation given a center and a scale factor.	Perform a dilation in the coordinate plane centered at the origin. Choose properties of a dilation given by a center and a scale factor. Choose a scale factor given a visual representation of a dilation and a center.	Determine the properties of a dilation given by a center and a scale factor. Perform a dilation in the coordinate plane given a scale factor and a center.	Illustrate a dilation and explain the properties between the images using precise mathematical language, given a pre-image, center, and scale factor.

Instructional Focus Statements

Level 3:

In grade 8, students are introduced to the foundation of dilations. They describe the effect of dilations on two-dimensional figures using coordinates. In high school geometry, students should extend their understanding of dilations by identifying and verifying properties that are formed between a pre-image and a dilated image. Students should use precise mathematical justification to explain that dilations preserve angle measure, betweenness, and collinearity.

Students should also be able to understand and demonstrate that dilations map a line segment (the pre-image) to another one segment whose length is the product of the scale factor and the length of the pre-image. Additionally, students should be able to understand and demonstrate that dilations map a line not passing through the center of dilation to a parallel line and leave a line passing through the center unchanged.

As students solidify their understanding of dilations they should be using a range of scale factors, including scale factors less and greater than 1 to gain a full understanding of how dilations can shrink as well as expand figures.

Level 4:

As students verify informally the properties of dilations, they should be able to work with a range of dilations where the scale factor is greater and less than 1 and a variety of centers of dilation. Students should be able to illustrate dilations using a variety of tools including but not limited to paper and pencil, dynamic geometry software, and the coordinate plane.

As students make connections to relate the sides of the image and pre-image of a dilation, they should be able to justify their reasoning using precise mathematical vocabulary.

Standard M2.G.SRT.A.2 (Major Work of the Grade)
Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Scope and Clarification:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Distinguish between a dilation and a translation, reflection, or rotation. Choose a dilation given a center and a scale factor. Choose two congruent triangles given all corresponding congruent angles and sides. Choose corresponding congruent sides and angles, given a triangle congruence statement.	Perform a dilation in the coordinate plane centered at the origin. Choose properties of a dilation given by a center and a scale factor. Choose a scale factor given a visual representation of a dilation and a center. Choose two similar triangles given all congruent corresponding angles and the proportionality of all corresponding pairs of sides. Choose congruent corresponding angles and the proportionality or all corresponding pairs of sides when given a triangle similarity statement Identify that two triangles are similar given that the triangles have been transformed by a single rigid	Determine the properties of a dilation given by a center and a scale factor. Perform a dilation in the coordinate plane given a scale factor and a center. Determine if two figures are similar using the definition of similarity in terms of similarity transformations. Identify which transformations preserve similarity for two triangles. Use similarity transformations to verify that all corresponding pairs of angles are congruent and verify the proportionality or all corresponding pairs of sides to show that the triangles are similar when given two triangles.	Illustrate a dilation and explain the properties between the images using precise mathematical language when given a pre-image, center, and scale factor. Explain the relationship between similarity transformations and the similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. Explain the definition of similarity both in terms of transformations and in terms of measurements of corresponding sides and angles. Create a pair of similar figures with a sequence of dilations and rigid motions and mathematically justify that the figures are similar.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	transformation followed by a dilation. Choose a scale factor given a simple dilation.	Determine the sequence of transformations that have occurred in order to determine when two figures are similar. Identify the ratio of proportionality for similar figures.	

Instructional Focus Statements

Level 3:

In grade 8, students began to develop a conceptual understanding of the effects of transformations on two-dimensional figures and use informal arguments to establish facts about angles. In the high school geometry standards, students use their prior understandings of transformations to discover the definition of similarity and further to discover which transformations preserve similarity in figures. In doing so, students should be given ample opportunity to explore sequences of transformations to visually conceptualize the relationship between the sides and the angles of similar figures. This should be done using tools that include but are not limited to patty paper, graph paper, and technology. Through this exploration, students should discover the definition of similarity in terms of similarity transformations and understand that similar figures have congruent pairs of corresponding angles and all pairs of corresponding sides have the same constant of proportionality. As students solidify their understanding of similar figures, they should understand that the scale factor increases or decreases the lengths proportionally while the angles remain congruent.

Level 4:

As students extend their understanding, they should be able work with a variety of sequences of dilations and rigid motions that determine if the image and pre-image are similar. Students should be able to explain the definition of similarity both in terms of transformations and in terms of measurements of corresponding sides and angles and explain the relationship using precise mathematical symbols, illustrations, and language. Additionally, students should be able to create a sequence of dilations and rigid motions to demonstrate a pair of similar figures and justify the similarity and the relationship that exists between the corresponding sides and angles.

Standard M2.G.SRT.A.3 (Major Work of the Grade)
Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Scope and Clarification:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify that two triangles are similar, given that two pairs of angles are congruent. Determine the third angle, given two angles in a triangle.	Determine if they are similar using the AA criterion, given two pairs of independently drawn triangles.	Determine if two triangles are similar or not similar by AA criterion using properties of similarity transformations.	Derive the AA criterion for similarity of triangles and explain the justification using precise mathematical vocabulary. Determine if two triangles are similar or not similar by AA, SSS, and SAS criteria, using properties of similarity transformations. Create similar triangles and prove that they are similar using the AA criterion using precise mathematical justification and vocabulary.

Instructional Focus Statements

Level 3:

Students began to develop a conceptual understanding of similar triangles in grade 8. In standard G.SRT.A.2, students gain an understanding that by using similarity transformations you can determine if two triangles are similar. Students grasped an understanding that the meaning of similarity for triangles is that all corresponding pairs of angles are congruent and all corresponding pairs of sides have the same constant of proportionality. This understanding, coupled with the angle sum theorem learned in the middle grades, results in students recognizing when pairs of corresponding angles are congruent, then the triangles are similar without verifying the proportionality of the side lengths, culminating in establishing the AA criterion.

Level 4:

As student gain an in-depth understanding of using the properties of similarity transformations to establish the AA similarity criterion for triangles, including SSS (side-side-side) and SAS (side-angle-side), they should be able to generalize and explain what measurements are needed to ensure that two triangles are similar. Students should also be able to provide illustrations with mathematical justifications, explaining which criteria is appropriate to prove similarity in which situations. Additionally, while the standards only explicitly states the AA similarity criterion, students should also discover other similarity criteria that work such as SSS (side-side-side) and SAS (side-angle-side), where the lengths of the corresponding pairs of sides are proportional.

Standard M2.G.SRT.B.4 (Major Work of the Grade)
Prove theorems about similar triangles.

Scope and Clarification:
Proving includes, but is not limited to, completing partial proofs; constructing two-column or paragraph proofs; using transformations to prove theorems; analyzing proofs; and critiquing completed proofs. Theorems include but are not limited to: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Complete a proportionality statement by choosing a missing side length, given two similar triangles that are formed by a line parallel to one side of the triangle. Choose the missing reason(s) that completes the proof, given a partial two-column proof.	Choose a proportionality statement, given two similar triangles that are formed by a line parallel to one side of the triangle. Choose the statement(s) and/or reason(s) that completes the proof, given a partial two-column proof. Choose the order in which the statements should appear, given a deconstructed paragraph proof.	Prove theorems about similar triangles by completing two-column and paragraph proofs. Use triangle similarity to prove the Pythagorean Theorem and its converse.	Determine and fix the error and explain the reasoning using precise mathematical justification, given a two-column and paragraph proof with errors. Prove theorems about similar triangles to discover that the geometric mean is the length of two parts of a right triangle when a perpendicular is dropped from the right angle to the hypotenuse.

Instructional Focus Statements

Level 3:

In this standard, students will apply their knowledge of similarity to prove various theorems about triangles. Students should have opportunities to explore situations and form conjectures that they can prove. In doing so, students should discover theorems, such as a line parallel to one side of a triangle divides the other two sides proportionally. Students should recognize that the line cuts the two sides proportionally and make the connection that the parallel lines form congruent corresponding angels and by the AA criteria the triangles are similar resulting in congruent angles and proportional side lengths. Also, students should use triangle similarity to prove the Pythagorean Theorem and its converse. These are explicitly stated in the scope and clarification of the standard but are not an exhaustive list.

Level 4:

As students extend their understanding of proving theorems about similar triangles, they should be able to work with increasingly challenging proofs and explain their reasoning in written and verbal formats. The standard's scope and clarification explicitly gives a few examples of theorems to prove but is not an exhaustive list. An additional example could include proving theorems about similar triangles to discover that the geometric mean is the length of two parts of a right triangle when a perpendicular is dropped from the right angle to the hypotenuse. To solidify understanding, students should be able to explain their reasoning by using appropriate mathematical symbols, vocabulary, and justifications.

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Standard M2.G.SRT.B.5 (Major Work of the Grade)
Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures.

Scope and Clarification:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose transformation relationships in simple geometric figures in cases where an image is not provided.	Choose transformation relationships in simple geometric figures in cases where an image is not provided.	Use transformations to determine relationships among simple geometric figures and to solve problems.	Apply congruence or similarity criteria to solve complex problems involving multiple concepts, and explain the geometric reasoning involved.

Instructional Focus Statements

Level 3:

Students expand their learning with proofs to a broader set of situations that may include triangle similarity or triangle congruence. Students should be able to work with a broad range of situations to provide arguments about their observations using triangle similarity and congruence to explain geometric relationships. As students form connections between pairs of similar and congruent triangles they should refer back to geometric transformations to explain how the triangles are related and establish the correspondence between the triangles. Students should also make prior course work connections and use triangle congruency criteria to determine properties of parallelograms and special parallelograms.

Level 4:

As students solidify their understanding of congruence and similarity criteria for triangles and how they are used to solve problems, they should extend their understanding by determining properties of parallelograms, special parallelograms, and other geometric figures. Students should be able to provide mathematical justifications explicitly linking similarity and congruence of triangles to other geometric figures. Additionally, students should be able to examine and critique justifications presented by others. This will help them develop more precise arguments of their own when solving real world problems.

Standard M2.G.SRT.C.6 (Major Work of the Grade)
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Scope and Clarification:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the side in relationship to a given angle in a right triangle. Identify the angle in relationship to a given side in a right triangle.	Know the definition of trigonometric ratios using right triangles. Choose the missing sides and angles of a right triangle, given other sides and angles.	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Find missing sides and angles of a right triangle, given other sides and angles.	Explain the relationship that exists between the ratios of the sides of right triangles that are proportional. Create right triangles that have and do not have proportional side lengths and explain the reasoning with precise mathematical justifications.

Instructional Focus Statements

Level 3:

Students should be able to understand that by similarity, side ratios in right triangles are properties of angles in the triangle. Students should use this understanding to explore and understand the definitions of trigonometric ratios for acute angles. The instructional focus is for students to have opportunities to explore different right triangles that have and do not have proportional side lengths to understand that corresponding sides of right triangles with the same acute angle measure will be proportional in length.

Level 4:

As students solidify their understanding, they should be able to explain why different right triangles that have and do not have proportional side lengths to understand that corresponding sides of right triangles with the same acute angle measure will be proportional in length. Students should also be able to justify when right triangles will be proportional in length. Student reasoning and justifications should encompass precise mathematical vocabulary.

Standard M2.G.SRT.C.7 (Major Work of the Grade)
Explain and use the relationship between the sine and cosine of complementary angles.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define sine and cosine ratios. Determine the measure of an angle in a right triangle by using the triangle angle sum theorem or the definition of complementary angles. Define complementary angles.	Draw a right triangle depicting given trigonometric ratios. Recognize complementary angles from a drawing or table. Recognize that the acute angles in a right triangle are complementary. Write a trigonometric ratio involving sine and cosine given a right triangle drawing. Make observations and identify patterns about trigonometric ratios from a table of values.	Develop logical arguments about relationships between the sine and cosine values of angles. Explain the relationship of the sine and cosine values of complementary angles. Use precise language to describe a trigonometric relationship. Use the concepts of the relationship between the sine and cosine values of complementary angles to solve non-routine problems such as complex drawings, embedded figures, or disseminating information that contains extraneous values.	Explore and draw conclusions about relationships of complementary angles using other trigonometric values. Critique the reasoning of others' explanation of the relationship between the sine and cosine of complementary angles. Create a real-world situation that would involve knowing the relationship between the sine and cosine of complementary angles.

Instructional Focus Statements

Level 3:

Similarity, right triangles, and trigonometry are grouped together to form a domain because it is necessary trigonometry is taught through the conceptual understanding of similarity. Trigonometry is one of the most widely used mathematical concepts and serves a variety of applications in a plethora of fields, so building the foundational knowledge of the special relationships in right triangles is crucial for students to make sense of trigonometry and apply trigonometric concepts fluently. This standard expands the work students do in grades 7 and 8. In grade 7, students explore relationships in scale drawings which leads to the study of similarity. In grade 8, students explore the effects of dilations. This lays the ground work for the definition of similarity and the special case of trigonometry. With that in mind, it is important for instruction to focus on providing students with the opportunity to notice, wonder, and conjecture about patterns in a table of sine and cosine values in a right triangle. There are many patterns to explore, and instruction should promote classroom discussion to make sense of the patterns, conjecture and test hypothesis, and justify true conjectures. Hence, modeling should be a focus of instruction. Particularly, the teacher should guide the exploration to discovering the relationship between the sine and cosine values of complementary angles. Students should be required to explain why the sine of an angle is equivalent to the cosine of its complementary angle and vice versa. Once students have developed conceptual understanding of the relationship, they should be able to use this concept to solve problems.

Instruction should hold students to a high standard of using precise language and explaining their reasoning with justification. This standard is one in a cluster where many relationships of right triangles are explored such as the definition of the trigonometric ratios and the Pythagorean Theorem. Students work with three trigonometric ratios in this course: tangent, sine, and cosine. It is imperative that students develop a conceptual understanding of the trigonometric ratios and the underpinning of trigonometry. Students should begin by solidifying the procedural fluency of labeling and accurately setting up trigonometric ratios to solve applied problems.

Level 4:

This standard serves as an excellent avenue for students to explore the beauty and phenomena of mathematics-particularly in trigonometric ratios. Challenge students to make sense of and conjecture about other patterns they find in a table of trigonometric values. Furthermore, instruction should provide ample opportunities for students to analyze and critique the thinking, reasoning, and arguments of others. Provide students the space to test other conjectures they find and try to prove or disprove them. Allow students the opportunity to generate and discuss real-world problems where knowing the sine and cosine values of complementary angles would be beneficial.

Standard M2.G.SRT.C.8 (Major Work of the Grade)
Solve triangles.
G.SRT.C.8a Know and use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.SRT.C.8b Know and use the Law of Sines and Law of Cosines to solve problems in real life situations. Recognize when it is appropriate to use each.

Scope and Clarification: (Modeling Standard)
Ambiguous cases will not be included in assessment.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Sketch and label the sides of right triangles. Use trigonometric ratios and the Pythagorean Theorem to choose the unknown side lengths of a right triangle.	Sketch, label, and identify the trigonometric ratios of a right triangle. Choose which trigonometric ratio is appropriate to use in solving a right triangle. Use trigonometric ratios and the Pythagorean Theorem to choose the unknown side lengths and angle measurements of a right triangle.	Use the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Law of Cosines to solve mathematical and real life problems and recognize when it is appropriate to use each.	Use the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Law of Cosines to solve complex mathematical and real life problems and recognize and explain when it is appropriate to use each.

Instructional Focus Statements

Level 3:

In grade 8, students develop an understanding of solving applied problems using the Pythagorean Theorem. As students encounter more complex applied problems and triangles that cannot be solved using the Pythagorean Theorem, they should develop an understanding of when it is appropriate to use the Pythagorean Theorem and when it is appropriate to use trigonometry.
Students work with three trigonometric ratios in this course: tangent, sine, and cosine. It is imperative that students develop a conceptual understanding of the trigonometric ratios and the underpinning of trigonometry. Students should begin by solidifying the procedural fluency of labeling and accurately setting up trigonometric ratios to solve applied problems.

Students should progress from using trigonometric ratios to using the Law of Sines and Law of Cosines. Students should encounter multiple examples where they develop a conceptual understanding of when it is appropriate to use the Pythagorean Theorem, right triangle trigonometry, and the Law of Sines and Law of Cosines. For any triangle, students should be able to solve for a specified angle or side and also be able to solve the triangle.

Level 4:

Students should solidify their procedural fluency around solving triangles using the Pythagorean Theorem, trigonometric ratios, and the Law of Sines and Cosines. They should be able to distinguish when it is appropriate to use each strategy and apply the strategy to solve applied problems. As students solidify their understanding, they should be able to explain their solution path and reasoning for their calculations, as well as their reasoning for selecting the appropriate strategy. Students should use precise mathematical vocabulary in their reasoning in written and verbal forms.

GEOMETRIC MEASUREMENT and DIMENSION (GMD)

Standard G.GMD.A.1 (Supporting Work)
Give an informal argument for the formulas for the circumference of a circle and the volume and surface area of a cylinder, cone, prism, and pyramid.

Scope and Clarification:
Informal arguments may include but are not limited to using the dissection argument, applying Cavalieri's principle, and constructing informal limit arguments. There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose and apply the formula for the circumference of a circle. Choose a statement that completes the argument, given a partial informal argument for the formula for the circumference of a circle. Apply the formula for the volume of a cylinder, cone, prism, and pyramid given the formula and a visual model.	Apply the formula for the circumference of a circle. Apply the formula for the volume of a cylinder, cone, prism, and pyramid given the formula. Choose a statement that completes the argument, given a partial informal argument for the formula for the volume or surface area of a cylinder, cone, prism, or pyramid.	Write an informal argument for the formulas for the circumference and area of a circle. Write an informal argument for the formulas for the volumes and surface areas of a cylinder, cone, prism, and pyramid.	Create a visual representation and write an informal argument for the formulas for the circumference and area of a circle. Create a visual representation and write an informal argument for formulas for volumes and surface areas of a cylinder, cone, prism, and pyramid.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

Students are applying geometric concepts learned in the middle grades in order to discuss why certain formulas work using informal arguments. The difference in this standard is that students should be able to justify why these formulas hold true.

Students began working with area in grade 3 and continued developing their conceptual understanding throughout the middle grades. This foundational understanding is the building block for students to develop a conceptual understanding of the surface area of geometric figures.

Students also began exploring circles in the early grades. The formula for the circumference of a circle can be seen when considering that, since all circles are similar, the ratios of their circumference to their diameter will be constant, defined as π. This is $\frac{C}{d}$ = π or C = d π, where C is the circumference and d is the diameter of the circle.

Students should also explore the formula for the area of a circle which is foundational for understanding the volume formula for a cylinder. Students can use the Cavalier's principle to extend their observation about the area of a circle to consider the volume of a cylinder.

Students should also make the connection that the volume of a pyramid is 1/3 the volume of a rectangular prism with the same base and height resulting in V = 1/3Bh, where B is the area of the base and h is the height of the prism. The same argument can be applied to a cone such that the volume of a cone is 1/3 of a cylinder with the same base and height resulting in the volume of a cone formula of V = 1/3 π r^2h, where r is the radius of the cylinder, and h is the height of the cylinder.

As students make these connections, they should be able to explain why the formulas work using drawings and models in written and verbal explanations. Central to this standard is students developing an understanding of why formulas look the way they do and an understanding of the origin of the formula as opposed to simply memorizing a formula for the sake of memorizing it.

Level 4:

As students solidify their understanding of why these formulas work using informal arguments, they should be able to explain the connections between the formulas. Students should be able to sketch drawings of geometric figures and explain the relationship between the different figures and how and why the volume formulas work. These justifications should be accompanied with the use of precise mathematical vocabulary.

Standard G.GMD.A.2 (Supporting Work)
Know and use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems.

Scope and Clarification: (Modeling Standard)
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Apply the formulas for the volume and surface area of a rectangular prism with integer dimensions given a visual model.	Apply the formulas for the volume and surface area of cylinders, cones, and spheres given the formula.	Apply volume and surface area formulas to solve mathematical and real-world problems.	Create real-world and mathematical problems and apply volume and surface area formulas to solve the problems.

Instructional Focus Statements

Level 3:

Students should use prior knowledge in order to know and use the volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems. Students should develop a conceptual understanding of applying the volume and surface area formula by making connections to the visual model and prior learnings. In standard G.GMD.A.1, students give informal arguments of volume and surface area formulas and their conceptual understandings. In this standard, students should use that developed conceptual understanding as a foundation to know the formulas and apply them to solve mathematical and real world problems.

Level 4:

As students solidify their understanding of surface area and volume formulas and their applications, they should begin the display procedural fluency in efficiently and accurately know formulas and using them in problems.

This standard directly addresses mathematical modeling. Given that geometric solids can be used to approximate many real life objects, the volume formulas can be used to address a broad range of contexts. Students should extend their learning to connect that not all real world scenarios can be perfectly modeled by geometric solids but it can provide approximation that yields useful information about the situation. Students should be able to explain this connection using precise mathematical vocabulary.

INTERPRETING CATEGORICAL and QUANTITATIVE DATA (S.ID)

Standard M2.S.ID.A.1 (Supporting Content)
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
M2.S.ID.A.1a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.

Scope and Clarifications:

i. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. Exponential functions are limited to those with domains in the integers.
ii. Tasks have a real-world context.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear function to fit a given data set. Choose if a given scatter plot is best represented by a linear, quadratic, or exponential function.	Choose a quadratic function that fits a given data set. Use a given linear function to solve a problems in the context of the data. Fit a linear function to a given set of data.	Fit a quadratic function to a given set of data. Fit an exponential function to a given set of data, where exponential functions are limited to domains in the integers. Solve problems using a linear, quadratic, and exponential function, where exponential function are limited to domains in the integers, in the context of the data.	Create a contextual situation with an embedded data set derived from a given function. Explain the relationship between the function, data set, and the contextual situation using precise mathematical language and justifications. Use a given function to explain the relationship between two quantities in a created context.

Instructional Focus Statements

Level 3:

In grade 8, students developed an understanding of how to create a scatterplot, evaluate the scatterplot in order to describe any pattern associations between the two quantities, and informally fit a straight line to data when it visually resembled a straight line. In high school, students should extend this understanding to summarize, represent, and interpret data on two categorical and quantitative variables. This allows students to use mathematical models to capture key elements of the relationship between the two variables and explain what the model tells about the relationship. Students should gain a conceptual understanding of how to draw conclusions in addition to finding the equation for the line of best fit. As students' progress through algebra it should become apparent to them that many real-world situations produce data that can be modeled using functions that are not linear. The exposure to quadratic and exponential functions broadens the options students have for modeling data sets, where data sets can be represented in tabular, graphical, or as a discrete set of points.

Students should be exposed to real-world situations where it is apparent that the scatter plot suggests a pattern that is more curved than linear in its visual depiction. Thus leading the student to realize that a linear function does not provide the closest fit to the data causing the student to consider other function types. It is imperative that students discover that sometimes obvious patterns may not tell the whole story. Students should develop an understanding that sometimes curves fit better than lines. Students should not only discover this algebraically but also develop an understanding of the connection that exists between the model and the contextual situation that it represents and understand that this connection is essential in identifying and building appropriate models. As students solidify their understanding, they should be able to describe how the variables are related within the context of the situation. Students should also use various forms of technology to explore and represent scatterplots as this will enhance their ability to see the relationship that exits between the variables.

Level 4:

As students extend their understanding, they should be able to create a contextual situation with an embedded data set derived from a given function. Students should also be able to explain and provide justifications for the relationships that exist between the function, data set, and the contextual situation using precise mathematical language. Particular attention should be put on creating situations that differentiate between linear, quadratic, and exponential functions. Students should be able to explain why one function is more appropriate than another function for the contextual situation.

Conditional Probability and the Rules of Probability (S.CP)

Standard M2.S.CP.A.1 (Supporting Content)
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. Identify the outcomes in a sample space which compose the event, for an event described in everyday language (e.g., "rolling double sixes").	Identify the union of two subsets, given a sample space within a context. Identify the intersection of two subsets, given a sample space within a context. Identify the complement of a subset, given a sample space within a context.	Represent sample spaces for compound events as unions, intersections, or complements of other events. Use the symbols ∪ and ∩ to represent sample spaces for compound events.	Create a Venn diagram to illustrate the union, intersection, or complement of events. Create a two-way frequency table to illustrate the union, intersection, or complement of events. Identify more complex subsets (e.g., the complement of $A \cup B$), given a sample space within a context.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

In grade 8, students represent sample spaces for compound events using lists, tables, and tree diagrams. In Algebra II, students are expected to expand their understanding of compound events by describing them as unions, intersections, or complements of other events. Teachers should build on students’ prior knowledge of compound events by using lists, tables, and tree diagrams to initially describe the union, intersection, and complement of events. Before moving on to contextual situations, teachers should ask students to find subsets of simple sets of numbers using set notation. Teachers can then relate set notation to different contextual situations, for example, a bag containing various colored marbles. As students become more comfortable with unions, intersections, and complements, they should be expected to describe compound events using the symbols ∪ and ∩. Using a Venn diagram can aid students in visualizing compound events. For example, let's assume there are 18 students enrolled in French (F), 45 students enrolled in Algebra II (A), 7 in both, and 114 enrolled in neither. How many students are there in $F \cup A$? Creating a Venn diagram and coloring in F, A, and the intersection would describe $F \cup A$.

In this situation, the union of French students and Algebra II students would be all those enrolled in French, Algebra II, or both. Students in both subjects should not be counted twice. Therefore, the intersection should be subtracted, and thus, there are 18 + 45 - 7 = 56 students in $F \cup A$. In A2.S.CP.B.6, students will build off of this idea to understand the addition rule with inclusive events. Teachers should remind students to not assume that the complement of F is A. The complement of F is all the students not in French, which would be the total (168) minus those enrolled in French (18), which is 150.

Although Venn diagrams are useful, situations with two categories are best described using a two-way table. For example, 65 students travel on a soccer trip. 43 are players and 12 are left handed. Only 5 of the left-handed students are soccer players. How many students are soccer players or are left handed?

Students should be expected to apply unions, intersections, and complements of events to calculating probability. Subsets can describe the number of ways an event can occur as well as the total number of possible outcomes. Therefore, a deep understanding of this standard sets the foundation for all other S.CP standards.

Level 4:

As students develop a deep understanding of this standard, they should be able to create their own Venn diagram or two-way table to describe various contextual situations. Therefore, students should be exposed to increasingly more complex real-world situations that are best described with a Venn diagram or two-way table. Additionally, students with a deep understanding of compound events can identify complex subsets. For example, students can find the complement of $A \cup B$ or see that the union of all subsets is the sample space.

Standard M2.S.CP.A.2 (Supporting Content)
Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Calculate the probability of two independent events occurring together, in a real-world context.	Calculate the joint probability of A and B occurring together, in a real-world context by examining the sample space. Identify whether theoretical probability events are independent or dependent (e.g., selecting two kings in a row with replacement versus without replacement).	Determine if the joint probability of A and B occurring together is equal to the product of their probabilities, and use this characterization to determine if they are independent.	Explain why two events are independent if the product of their probabilities is equal to the joint probability.

Instructional Focus Statements

Level 3:

In grade 8, students find probabilities of compound events. In Algebra II, students are exposed to both independent and dependent events when calculating joint probabilities. One misconception students have is assuming P(A) and P(B) are independent and thus, they simply multiply P(A)*P(B) without considering if the likelihood of one event occurring affects the probability of the other event occurring. Students should be able to decide if given events are independent or dependent. For example, find the probability of selecting a blue marble from a bag and then selecting a red marble from the same bag after placing the first marble selected back into the bag. This is independent because the probability the first marble is blue does not impact the probability of the second marble being red. Teachers should expose students to events of both types to strengthen their ability to categorize various situations as independent or dependent (e.g., also give students an example where the first marble is not replaced). If students decide the given events are independent, then P(A and B) = P(A)*P(B). Conversely, if P(A and B) = P(A)*P(B), then students can discern that the events are independent.

Many students take advantage of the first statement, but do not immediately understand the second statement of the bi-conditional. Therefore, students should be given opportunities to examine the sample space to decide if the events are independent. For example, let's assume there two light switches. If the light switches are randomly assigned to on or off, what is the probability that the first one is off and the second is on? Students should be able to write the sample space of all the possibilities, which includes OO, OF, FO, and FF (O is on and F is off). In this scenario, students should see that only one of the cases is FO and thus, the probability is 1/4. Since 1/4 is 1/2 times 1/2, then it is equivalent to P(F)*P(O). Therefore, students can decide that the two events are independent and explain how it makes sense because the probability of the second light being on is not impacted by the probability of the first light being off, because they were randomly assigned.

If events are dependent, then students must first consider how the probability of the first event impacts the probability of the second event. The product changes to P(A and B) = P(A)*(B given A), which uses the conditional probability. Instead of memorizing the formula, students should reason and make sense of the problem and how the first event impacts the second. In fact, the formula for joint probabilities is always P(A and B) = P(A)*P(B given A), but with independent events, P(B given A) is the same as P(B) because P(A) does not impact P(B). This idea leads nicely into conditional probability and independence. Thus, instruction can be paired nicely with M2.S.CP.A.3 and M2.S.CP.A.4, in which students will calculate conditional probabilities and use them to decide independence.

Level 4:

As students develop a deep understanding of independent and dependent events, they should be expected to explain why two events are independent if the product of their probabilities is equal to the joint probability. Students can provide an example like light switch problem in Instructional Focus level 3 and explain how if the product of their probabilities is not equal to the joint probability, then the probability of the second event is altered by the probability of the first event, and thus the events are dependent.

Standard M2.S.CP.A.3 (Supporting Content)
Know and understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

Scope and Clarifications:
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Calculate the probability of two independent events occurring together, in a real-world context. Recognize conditional probabilities in a real-world context.	Calculate conditional probabilities in a real-world context. Calculate the probability of two dependent events occurring together, in a real-world context.	Explain why events A and B are independent if the probability of B given A is the same as the probability of B. Explain why the probability of B given A is the same as the probability of B if events A and B are independent.	Explain why the probability of A given B is P(A and B)/P(B).

Instructional Focus Statements

Level 3:

Understanding the bi-conditional statement connecting independence to P(B) = P(B given A) continues in this standard as students work with conditional probability. While students develop an understanding of A2.S.CP.A.2, they should begin to understand how to calculate probabilities when the probability of the second event (B) is affected by the probability of the first event (A). By calculating the new probability of B, given that A has already occurred, students are discovering the idea of conditional probability. A2.S.CP.A.3 builds upon A2.S.CP.A.2 to support students in developing a conceptual understanding of conditional probability and why the conditional probability of A given B is P(A and B)/P(B). One way to accomplish this is to begin with the formula for dependent probability, P(A and B) = P(A)*P(B given A). Students can divide both sides of the equation by P(A) to rearrange the formula to reveal P(B given A) = P(A and B)/P(A). Counting methods such as the counting principle, tree diagrams, tables, and lists can be used to help students conceptually understand this principle.

For example, an academic club at school consists of 4 freshman, 6 sophomores, 9 juniors, and 11 seniors. Two students are selected from the club to join the school parade. What is the probability that the second student selected is a junior given the first student selected was a senior? In this scenario, students can simply count how many juniors and total students remain once the senior is selected. This would leave 9 juniors and 19 total students, and thus, P(Junior given Senior has already been selected) would be 9/19. However, students can make the connection by dividing the probability of selecting a senior and junior by the probability of selecting a senior. The counting principle would give P(Senior and Junior)/P(Senior) = $\frac{(11/20)*(9/19)}{(11/20)} = \frac{9}{19}$. In this situation, students can calculate the probability both ways and notice that P(Junior given Senior has already been selected) = P(Senior and Junior)/P(Senior).

However, students aren't always able to write P(A and B) separately as P(A)*P(B), so the formula can be beneficial in those situations. For example, the probability that a randomly selected car has cruise control is 92%. The probability that a randomly selected car has both cruise control and a rear camera is 45%. What is the probability that a car with cruise control also has a rear camera, P(Camera given cruise control)? Students should use the formula to produce $\frac{.45}{.92} = .489$. Understanding the conditional probability B given A as the fraction of A's outcomes that also belong to B is the focus of A2.S.CP.B.5, so these standards can be nicely paired together.

Level 4:

Explaining why the probability of A given B is P(A and B)/P(B) can take many forms. Students with a deep understanding of this standard should be able to use Venn diagrams, algebra, and contextual situations to explain why the probability of A given B is P(A and B)/P(B). They should be able to describe contextual situations and how conditional probability can be applied to solve problems.

Standard M2.S.CP.A.4 (Supporting Content)
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Scope and Clarifications:
For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Complete a hypothetical 1000 two-way table, given the probability of A, the probability of B, and the probability of A and B.	Calculate conditional probabilities within a context from a two-way table.	Compare conditional probabilities from a two-way table to decide if the events are independent or not independent. Explain why two conditional probabilities from a two-way table with the same conditions means the two events are independent.	Create a real-world context and corresponding two-way table in which the events are independent.

Instructional Focus Statements

Level 3:

Two events are independent when knowing that one event has occurred does not change the likelihood that the second event has occurred. Students can use conditional probabilities to tell if two events are independent or not independent from a two-way table. For example, the probability that a student knows what they want to do after high school is 75%, the probability that a student attends extracurricular activities is 40%, and the probability that a student does not know what they want to do after high school and attends extracurricular activities is 30%. Are the events "a student knows what they want to do after high school" and "attends extracurricular activities" independent or not? One way to determine independence is to compare P(extracurricular activities given they know what they want to do after high school), which is P(A and B)/P(A) = 30%/75% = 40% with P(extracurricular activities), which is 40%. In this case, P(B given A) = P(B), therefore, the two events are independent.

It is also helpful for students to create a hypothetical 1000 two-way table to easily organize the information in rows and columns. In the two-way table, students would use 300 instead of 30%, for example. If the probability of attending extracurricular activities given the student knows what they want to do after high school (300/750 = 40%) is equivalent to the probability of attending extracurricular activities given the student does not know what they want to do after high school (100/250 = 40%), then the two events are independent. This should make sense to students because that would mean that the probability of attending extracurricular activities is the same regardless of whether the student knows what they want to do after high school or not. If the probabilities were not equal or relatively close, then students cannot assume the events are dependent, only not independent. Although there may be an association between the two variables, teachers should remind students that association does not mean there is a causal relationship.

Level 4:

As students develop a deep understanding of determining independence by comparing two conditional probabilities with the same condition, they should be able to create their own real-life context with data and discern whether the two events are independent or not independent. Is can be helpful for some students to create data that results in the events being not independent so that other students can be exposed to those situations when examples are shared with the class.

Standard M2.S.CP.B.5 (Supporting Content)
Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

Scope and Clarifications:
For example, a teacher gave two exams. 75 percent passed the first quiz and 25 percent passed both. What percent who passed the first quiz also passed the second quiz? There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize conditional probabilities in a real-world context. Determine what percent a given percent is of another percent (e.g., 5% is what percent of 20%?). Divide two fractions and put into simplest form.	Calculate conditional probabilities in a real-world context. Recognize the conditional whole in contextual problems is not the total.	Use a model (e.g., Venn diagram or table) to represent and solve conditional probability problems.	Explain the conditional probability of A given B as the area of A and B divided by the area of B using a Venn diagram.

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

Based on other conditional probability standards, students should recognize that the probabilities P(B) and P(B given A) are different if the two events are not independent. In A2.S.CP.A.3, students develop a conceptual understanding of why P(B given A) = P(A and B)/P(A). In this standard students use this formula to calculate conditional probabilities (i.e., P(B given A)) in various real-world situations. In many cases, students are given the P(A and B) and either P(A), P(B), or both and asked to calculate the conditional probability P(B given A) or P(A given B). Discussion during instruction would need to help students recognize that if they are given both P(A) and P(B), one is superfluous information. It can be helpful to model the probability using a Venn diagram to visualize a percent of a percent. For example, given the probability that a randomly selected student is a female is 56% (A), the probability the student is in drama is 67% (B), and the probability that the student is female and in drama class is 34% (A and B), what is the probability the student is in drama given they are female (B given A)? Since students are given the selected student is female, the total is narrowed to only female students (56%) instead of 100%. Instruction should lead students to recognize that the question is asking what percent of females are in drama which is, mathematically, finding what percent of 56% is 34%. This would lead students to recognize they need to divide .34/.56, which is P(A and B)/P(A). A Venn diagram can depict the idea that conditional probability is the intersection, P(A and B), divided by the conditional whole, in this case, P(A).

If more than two categories are used, teachers can provide or encourage students to use a table. For example, a frequency table can be used to compare the interest in sports across grade levels. If students are asked to calculate the probability a randomly selected student likes sports given they are in the 10th grade, students could be shown how to cover up the rest of the table to highlight only 10th graders. From there, students can calculate the probability that a student likes sports. Explaining that the condition given narrowed down the subjects to only 10th graders helps students begin to focus on only the values involved in the problem. As students use a table, they can further strengthen their understanding of calculating a percent of a conditional whole.

Level 4:

As students use the conditional probability formula and begin to see conditional probability as a percent of a percent, they should be expected to explain conditional probability using a Venn diagram or table. Teachers should ask students to discuss how the whole is narrowed to a conditional whole and how that impacts the probability of a given event. Using a Venn diagram, students can shade in the area of A and B and show how it is a fraction of the whole (either A or B). Likewise, students could be asked to justify why conditional probability narrows a table to a row or column, in which the probability can then be found. Teachers can ask students to provide an example and write a response about their findings, explaining how the conditional probability was calculated and justifying their thinking. This activity would support the literacy standards in mathematics, as students would be discussing and articulating mathematical ideas while using correct mathematical vocabulary to build a their argument.

Standard M2.S.CP.B.6 (Supporting Content)
Know and apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Scope and Clarifications:
For example, in a math class of 32 students, 14 are boys and 18 are girls. On a unit test 6 boys and 5 girls made an A. If a student is chosen at random from a class, what is the probability of choosing a girl or an A student?
There are no assessment limits for this standard. The entire standard is assessed in this course.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. Find P(A or B), given two events that are mutually exclusive. Add and subtract fractions and put into simplest form.	Know that P(A or B) = P(A) + P(B) - P(A and B) for both inclusive and mutually exclusive events. Determine if two events are inclusive or mutually exclusive. Identify the union of two subsets, given a sample space within a context. Identify the intersection of two subsets, given a sample space within a context.	Calculate P(A or B) given two events within a real-world context. Interpret P(A or B) in terms of the real-world context.	Explain why P(A or B) = P(A) + P(B) - P(A and B) using a model (e.g., Venn diagram or table). Explain why P(A or B) = P(A) + P(B and the complement of A) is an alternate solution method. Explain why P(A or B) = P(A and the complement of B) + P(B and the complement of A) + P(A and B) is an alternate solution method.

Instructional Focus Statements

Level 3:

In grade 8, student calculate the probability of compound events by examining the sample space using lists, tables, and tree diagrams. In Integrated Math II, students use their prior knowledge from grade 8 and their understanding of subsets formed in A2.S.CP.A.1 to calculate P(A or B), when A and B are inclusive or mutually exclusive. One misconception students tend to have is that "or" always means add, and therefore, simply calculate P(A or B) as P(A) + P(B). Teachers can break this misconception by giving students examples of inclusive events where P(A) + P(B) is greater than 1. For example, when rolling a 6-sided number cube, what is the probability of rolling an odd number or a number greater than 2? In this case, P(A) + P(B) = 3/6 + 4/6 = 7/6. Discussion should help students recognize that this cannot be correct and realize that by simply adding the probabilities together, they counted 3 and 5 twice. Having students examine the sample space can quickly reveal what the probability should be. At this point, teachers can give students time to discover how to calculate P(A or B) by starting with P(A) + P(B) = 7/6 and asking students to identify the overlap and see that they need to subtract it to calculate the probability correctly. This should help students understand the need to subtract the outcomes that fit in both categories and were counted twice. Thus, students calculate P(A or B) as 7/6 - 2/6 = 5/6.

Instead of giving students the formula, teachers should let students discover why the P(A and B) needs to be subtracted out so that outcomes (i.e., P(A and B)) are not counted twice. It can be helpful to use a Venn diagram so that students can visualize this concept and make the connection to the formula, P(A or B) = P(A) + P(B) - P(A and B). If A and B are inclusive, shading P(A) and P(B) on a Venn diagram reveals to students how the intersection is being shaded twice.

Level 4:

A Venn diagram or two-way table can support students in developing a deep understanding of this standard. Instruction should encourage students to explain why P(A or B) = P(A) + P(B) - P(A and B) using multiple representations, such as both a Venn diagram and a two-way table. By having students create a two-way table, the P(A and B) would be separated out from P(A) and P(B) and students can see these four probabilities shown in the table: P(A and the complement of B), P(B and the complement of A), P(A and B), and P(the complement of B and the complement of A). By displaying data in this form, students should see other ways to calculate P(A or B), such as P(A) + P(B and the complement of A), or P(A and the complement of B) + P(B and the complement of A) + P(A and B). Asking students to separate a Venn diagram into its four parts can also aid in understanding these alternate ways to calculate P(A or B) and often better aligns with students' thinking as they examine the sample space to calculate P(A or B).