

## **Algebra I Instructional Focus Documents**

### Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

### Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 4: Performance at this level demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in this same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting.

### Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

## Quantities (N.Q)

### Standard A1.N.Q.A.1 (Supporting Content)

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

### Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose units appropriately when solving a simple problem.</p> <p>Choose a graphical representation to represent a real-world problem.</p> <p>Choose a data display that describes the values and units in a problem.</p>	<p>Choose and interpret units appropriately when solving a simple problem.</p> <p>Identify when units need to be converted to the same unit within a contextual problem.</p> <p>Connect values to the units to represent given information.</p> <p>Choose appropriate units in order to evaluate a formula, given an input value.</p> <p>Choose an interpretation of the graph that represent a real-world problem.</p> <p>Choose a data display with</p>	<p>Use units as a way to understand problems and to guide the solution path for a multi-step problem.</p> <p>Choose and interpret units appropriately when solving a multi-step problem, including problems that contain real-world formulas.</p> <p>Recognize the relationship between the units for all variables in a formula.</p> <p>Choose and interpret the scale and the origin in graphs and data displays.</p> <p>Determine the most appropriate data display based on the units given in a problem.</p>	<p>Explain if the information is represented appropriately using mathematical justification, given a numerical and/or a graphical representation of a real-world problem</p> <p>Create a real-world problem involving formulas and data represented either in a table or graph in which the data must be analyzed for appropriate units and scale. Explain the interpretation of the units, scale, and origin with respect to the contextual situation using precise mathematical vocabulary.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	appropriate units that represent a real-world problem.		

### Instructional Focus Statements

#### Level 3:

Instruction for this standard should focus on the importance of correctly interpreting units and representing contextual problems using appropriate data displays. Students should explore the importance of correctly interpreting units and designing appropriate displays of data to most appropriately represent a contextual problem. Instruction should be centered around tasks that provide real-world, multi-step problems where interpreting units appropriately is critical. Students should be engaged in classroom discourse (MP 8) that promotes explaining their reasoning and justifying quantities as a result of a solution pathway. Students should be given ample opportunity to work with data representations where students have to think critically to set an appropriate scale that displays the data to show key features. This standard is important throughout Algebra 1, Geometry, and Algebra 2 as modeling real-world problems should be prevalent in all three courses. It is imperative that students attend to precision in using, interpreting, and reporting units. This is a standard that students will continue to utilize throughout high school. Students should understand that a key relationship exists between units and appropriate representation of units and this understanding is beneficial to develop a conceptual understanding of units in contextual problems. This standard should be integrated within classroom instruction throughout the year, and students should apply it in descriptive modeling.

#### Level 4:

One extension of this standard is for students to differentiate between data that is appropriately displayed versus data that is not appropriately displayed. Students can then analyze the difference between both displays providing a critique of the representation. This should be done using logical arguments, including explaining how to improve the representation so that all key features and units are displayed appropriately. Data should be represented graphically, numerically, algebraically, and verbally. Students should be able to interpret, solve, and represent contextual information using appropriate units. Students at this level should attend to precision when interpreting and using units. Furthermore, students should be given opportunities to synthesize information from multiple sources and produce a descriptive model that represents the contextual situation.

**Standard A1.N.Q.A.2 (Supporting Content)**

Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.

**Scope and Clarifications: (Modeling Standard)**

Descriptive modeling refers to understanding and interpreting graphs; identifying extraneous information; choosing appropriate units; etc.

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

<b>Students with a level 1 understanding of this standard will most likely be able to:</b>	<b>Students with a level 2 understanding of this standard will most likely be able to:</b>	<b>Students with a level 3 understanding of this standard will most likely be able to:</b>	<b>Students with a level 4 understanding of this standard will most likely be able to:</b>
<p>Identify the units in a problem.</p> <p>Connect the units to the values in a real-world problem.</p>	<p>Identify individual quantities in context of the real-world problem and label them with appropriate units.</p> <p>Determine if quantities are labeled with the correct units in the context of a real-world problem.</p> <p>Recognize extraneous information in a real-world problem.</p>	<p>Identify and interpret necessary information in order to select or create a quantity that models a real-world problem.</p> <p>Explain the meaning of individual quantities in the context of the real-world problem.</p> <p>Attend to precision when defining quantities and their units embedded in context.</p> <p>Explain and justify the relationship between solutions to contextual problems and the values used to compute the solutions.</p> <p>Appropriately interpret, explain the meaning of, and draw conclusions about the quantities in a real-world problems.</p>	<p>Identify, interpret, and justify complex information embedded in a real-word problem containing a variety of descriptors or units in order to solve contextual problems for the purpose of descriptive modeling.</p> <p>Represent quantities in descriptive modeling situations and explain their relationship using numeric, algebraic, and graphical representations.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Make observations about quantities given a graph or model. Explain why information is extraneous in a real-world problem.	

### Instructional Focus Statements

**Level 3:**  
 In grades K-8, students developed an understanding of measuring, labeling values, and understanding how the value of a number relates to the described quantity. In the high school Numbers and Quantity (NQ) domain, students develop an understanding of reasoning quantitatively and solving problems requiring the evaluation of the appropriateness of the form in which quantities are provided. Instruction for this standard should be integrated with a wide variety of standards throughout the course. Students should extend their understanding of using appropriate quantities in descriptive modeling situations where they can make comparisons between two distinct quantities and justify the quantities appropriately in order to describe or to solve a contextual problem. Descriptive modeling refers to understanding and interpreting graphs, identifying extraneous information, choosing appropriate units, etc. Instruction should focus on providing opportunities for students to select appropriate quantities embedded in real-world contextual problems and attend to precision by describing the quantities in descriptive modeling situations. The study of dimensional analysis is an excellent avenue to help students understand how critical values, units, and quantities are used in interpreting information and modeling a real-world problem. Furthermore, students must be given opportunities to write and create appropriate labels for quantities and explain the meaning of the quantities in a context. Being able to identify, interpret, and justify quantities is a skill that will serve students well to have mastered during this course as this standard lays the foundation for using units as a way to understand problems.

**Level 4:**  
 Instruction should focus on providing opportunities for students to work with problems that have a variety of descriptors and units embedded in the context. Students should be asked to extend their knowledge of quantities by representing them in multiple formats such as a graphical representation of the given information, algebraic representation of the quantities, and multiple representations to predict or draw conclusions about the solution of the real-world problem. Instruction should provide opportunities for students to analyze and critique the interpretation of quantities in a descriptive modeling problem. Additionally, students should be given ample opportunities to design their own contextual problem in which they would have to use quantities appropriately in order to describe the modeled contextual situation.

**Standard A1.N.Q.A.3 (Supporting Content)**

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Scope and Clarifications: (Modeling Standard)**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Explain the difference between precision and accuracy.</p>	<p>Choose a solution that is both accurate and reasonable with respect to the contextual situation.</p> <p>Describe how using inaccurate measurements when reporting quantities can affect the solution.</p>	<p>Report a quantity with precision and accuracy.</p> <p>Choose an appropriate level of accuracy that reflects the limitations on measurement. Explain the reasonableness of answers with respect to the context of the problem when reporting quantities as a result of solving the contextual problem.</p> <p>Describe the most common causes of inaccuracies in contextual problems (e.g., when using measurement tools).</p>	<p>Describe the accuracy of a measurement embedded in a real-world context by stating the possible error when appropriate.</p> <p>Explain why it is important to choose an appropriate level of accuracy and what limitations exist on measurement when reporting quantities in contextual problems.</p>

**Instructional Focus Statements**

**Level 3:**

This standard builds upon the opportunities students have been given to explain values of numbers in terms of units in previous grades. In middle school, students focused on ratios and proportional relationships and also using quantities to describe data from a statistical lens. Both fields help prepare

students for choosing a level of accuracy when reporting quantities. Additionally, students have had experience interpreting and reporting quantities that involve area, volume, and rates.

As this is a modeling standard, students should solve contextual problems and be able to choose a level of accuracy of measurement quantities that is reasonable and makes sense to the contextual situation. For example, when solving a multi-step problem or using graphing technology, students should determine when it is appropriate and not appropriate to use precise values (values that are not rounded or truncated), rounded values, or truncated values. Students should be able to justify their reasoning for using certain values and explain why their choice is important with respect to the context. Additionally, students should experience different solution paths that involve using different forms of values and explain how accuracy does or does not have an impact on the solution within the context of the problem.

Instruction should focus on providing students a plethora of opportunities to use a variety of measurements including measuring tools and graphing technology. Students should have ample time to explore traditional, physical tools as well as electronic, and digital tools. During this exploration, class discussion should focus on helping students ascertain the difference between precision and accuracy and when it is appropriate to apply each of them in certain problems. Furthermore, instruction should be infused with a broad spectrum of different types of units that describe tiny to very large quantities. This is a modeling standard and students should make connections to other disciplines such as science.

#### **Level 4:**

Students have a great opportunity to support their understanding of this standard through the lens of a wide variety of other disciplines. Instruction should focus on providing students with experiences involving problem situations that interest them. Include inquiry with this standard and allow students ample time to explore repeated measurement in order to determine an acceptable level of accuracy when reporting quantities. Also, instruction should provide the opportunity for students to analyze and critique the level of accuracy chosen by others to report quantities.

This modeling standard is a great way to make connections to other disciplines, specifically science. An extension of this standard can include applying the concept of significant figures, especially in science related contexts. Additionally, in science contexts, students can apply their knowledge of significant digits and scientific notation to explore tasks and report quantities appropriately.

## SEEING STRUCTURE in EXPRESSIONS (A.SSE)

### Standard A1.A.SSE.A.1 (Major Work of the Grade)

Interpret expressions that represent a quantity in terms of its context.

**A1.A.SSE.A.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**A1.A.SSE.A.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

### Scope and Clarifications: (Modeling Standard)

For example, interpret  $P(1 + r)^n$  as the product of  $P$  and a factor not depending on  $P$ .

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify parts of an expression (i.e. factor, coefficient, term).</p> <p>Define the formal definition of the terms: factor, coefficient, and term.</p> <p>Define the formal definition of the term expression.</p> <p>Label the single entities in an expression.</p>	<p>Recognize arithmetic operations in an expression in order to see the structure of the expression.</p> <p>Understand and use the definitions of terms, factors, coefficients, and like terms in order to describe the structure of the individual parts of the expression.</p> <p>Identify parts of an expression as a single entity.</p> <p>Recognize that individual parts of an expression affect the whole expression.</p> <p>State arithmetic operations performed within an expression.</p>	<p>Interpret parts of an expression (i.e. term, factor, coefficient) embedded in a real-world situation and explain each part in terms of the context.</p> <p>Interpret parts of an expression (i.e. term, factor, and coefficient) and explain each part in terms of the function the expression defines.</p> <p>Explain the structure of an expression and how each term is related to the other terms by interpreting the arithmetic meaning of each term in the expression and recognizing when combining like terms is appropriate.</p> <p>Interpret an expression by describing each individual term as a</p>	<p>Interpret expressions in a variety of forms by explaining the relationship between the terms and the structure of the expression.</p> <p>Interpret parts of complex expressions with varying combinations of arithmetic operations and exponents by viewing one or more of their parts as a single entity.</p> <p>Write and interpret expressions that represent a real-world context and use the expressions to solve contextual problems.</p> <p>Write expressions in a wide variety of formats and then for each</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		single entity and the relationship to the expression.	<p>describe the effects each term has considering them first individually and then considering them as a part of the expression.</p> <p>Identify and explain structure in patterns represented pictorially or graphically and write an algebraic expression to represent the pattern.</p>

### Instructional Focus Statements

**Level 3:**

Seeing structure in expressions is the connecting bridge between arithmetic operations in grades K-8 and algebraic thinking in high school. Instruction should build on students understanding of the relationship between arithmetic operations in expressions and equations. Students should explore a variety of expressions in equivalent forms to see and evaluate the structure in each form. Students should be exposed to exponents of varying degree. This allows them to recognize the attributes of a term in order to combine it appropriately with other like terms. Instruction should expose students to a variety of multiple representations and require students to interpret and explain the relationship between the representations. Students should be challenged with complex, multi-variable expressions to interpret.

Furthermore, students must be able to explain individual terms and interpret that term as a single entity and as a whole expression. Instruction should focus on using the structure of the expression to uncover the attributes of the function it defines. Students should also be able to use precise language to explain the relationship between a verbal description and an algebraic representation. Particular focus needs to be placed on translating words into mathematical expressions and vice versa.

**Level 4:**

Students need to be presented with complex expressions that include a combination of different arithmetic operations and interpret in terms of a real-world context. The pinnacle of level 4 understanding is being able understand, interpret, and explain the relationship between equivalent representations

of an expression. Students should be able to explain not only the expression in terms of a contextual situation, but also how each term within the expression connects back to the contextual situation.

Additionally, instruction should focus on relating expressions to real world contexts. For example, students should be given problems that describe contextual situations from multiple perspectives. Students should interpret the contextual situation for each individual perspective and write an expression that represents the context for each. Students should be challenged to interpret the meaning of the expressions created and use them to predict outcomes and solve problems. Instruction should expose students to multiple representations of the expressions by making connections between the equivalent expressions, which will in turn help students recognize the most useful form of an expression depending on context. Students should be challenged to justify why other formats are equivalent and which format is most relevant given the context of the problem.

**Standard A1.A.SSE.A.2 (Major Work of the Grade)**

Use the structure of an expression to identify ways to rewrite it.

**Scope and Clarifications:**

For example, recognize  $53^2 - 47^2$  as a difference of squares and see an opportunity to rewrite it in the easier to evaluate form  $(53 + 47)(53 - 47)$ . See an opportunity to rewrite  $a^2 + 9a + 14$  as  $(a + 7)(a + 2)$ . Tasks are limited to numerical expressions and polynomial expressions.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a numerical or polynomial expression that is equivalent to a given expression.	Rewrite numerical and polynomial expressions into a given form.	Rewrite numerical and polynomial expressions in a different form and explain why rewriting the expression in that form is beneficial.	Generate multiple forms of a single numerical or polynomial expression and explain in both verbal and written form the mathematical reasoning that was employed to rewrite the expression.  Additionally, explain which form is most useful and provide mathematical justification.

**Instructional Focus Statements**

**Level 3:**

In grade 7, students developed an understanding of using properties to generate equivalent linear expressions. Additionally, they extended this understanding to identify when certain forms of an expression are more useful than others.

In Algebra I, students should continue developing the idea that there are often multiple ways to write expressions. Students need to be able to see complicated expressions as built from simpler ones.

Students should be able to provide a mathematical justification for when different forms of expressions are more beneficial. Particular focus needs to be placed on quadratic polynomial expressions as they are a focus of this course. Much of the ability to see and use structure in transforming expressions comes from learning to fluently recognize certain fundamental algebraic situations.

Developing procedural fluency with quadratics will serve as a cornerstone for future course work and will extend into future courses work with higher powered polynomials.

**Level 4:**

Students need to be challenged to write polynomial and numerical expressions in multiple forms where the initial expressions increase in difficulty over time. The hallmark of this standard is students being able to communicate the importance and benefit gained from writing expressions in various forms. Students should be able to express what the individual terms within the expression mean and how they relate to terms in the other various representations of the same expression.

**Standard A1.A.SSE.B.3 (Major Work of the Grade)**

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

**A1.A.SSE.B.3a** Factor a quadratic expression to reveal the zeros of the function it defines.

**A1.A.SSE.B.3b** Complete the square in a quadratic expression in the form  $Ax^2 + Bx + C$  to reveal the maximum of the function it defines.

**A1.A.SSE.B.3c** Use the properties of exponents to rewrite exponential expressions.

**Scope and Clarifications: (Modeling Standard)**

**For A1.A.SSE.B.3c:**

For example, the growth of bacteria can be modeled by either  $f(t) = 3^{(t+2)}$  or  $g(t) = 9^{(3t)}$  because the expression  $3^{(t+2)}$  can be rewritten as  $(3^t)(3^2) = 9(3^t)$ .

- i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.
- ii) Tasks are limited to exponential expressions with integer exponents.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose the correct factorization when given a quadratic expression where <math>A=1</math>.</p> <p>Determine if the function will have a maximum or a minimum, given a quadratic expression in the form <math>Ax^2 + Bx + C</math>.</p> <p>Identify which expressions are exponential from a list of expressions representing various function families.</p> <p>Identify which represent true properties of exponents, from a list</p>	<p>Choose the zeros of the function it defines, given the factorization of a quadratic expression.</p> <p>Choose the maximum or minimum value of a function defined by a quadratic expression in vertex form <math>(x-p)^2 = q</math>.</p> <p>Choose an equivalent form of an exponential expression and choose the properties used to transform the expression, given a real-world context.</p>	<p>Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>Identify equivalent forms of quadratic expressions.</p> <p>Determine the maximum or minimum value of a function defined by a quadratic expression in the form <math>Ax^2 + Bx + C</math> by completing the square.</p> <p>Generate an equivalent form of the exponential expression and identify the properties of exponents used to generate the equivalent expression,</p>	<p>Explain the most efficient method for factoring a given quadratic expression and justify the reasoning.</p> <p>Recognize algebraically and graphically when a quadratic has 0, 1, or 2 x-intercepts.</p> <p>Determine the maximum or minimum value of a function defined by a quadratic expression by completing the square embedded in a real-world situation and explain the x and y-coordinates of the maximum or minimum within</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>of properties (some true and some not true).</p> <p>Choose an equivalent form of a given exponential expression not embedded in a real-world situation.</p>		<p>for an exponential expression embedded in a real-world context.</p>	<p>the context of the real-world situation.</p> <p>Generate equivalent forms of an exponential expression embedded in a real-world situation, justify each transformation with a property, and explain the benefits of the equivalent expression in terms of the context of the problem.</p>

### Instructional Focus Statements

#### **Level 3:**

Students should be able to factor quadratic expressions using various methods. Student understanding of factoring should draw on connecting how and why a quadratic expression and a factored form of the same expression are equivalent. It is important that students gain more than a rote procedural understanding of the process of factoring across all standards involving factoring in Algebra I. They need to conceptually understand how factoring works and why factoring works. That said, the primary focus of this standard is connecting the factored form graphically to the zeroes of the function the quadratic expression represents. Students are developing an understanding of when and why the factored form is helpful. It is imperative that they connect the terms “zeros” and “x-intercepts” as one is typically geared towards expressions and equations and the other towards graphs. Emphasis should be given to using the factored form of an expression as a means to identify the zeros of the function and then tying that to a graphical representation.

In conjunction with developing proficiency in factoring quadratic expressions, student should be developing an understanding of the value of completing the square. Students need to develop an understanding of when each technique is beneficial along with an understanding of why each is beneficial. Students should realize that completing the square is a method for producing an equivalent form of a quadratic expression which reveals the maximum or minimum of the function defined by the expression. Once again it is important that students connect the quadratic expression and the graph of the function defined by the expression. It is important that there be strong instructional connection with this standard and A1.F.IF.C.6. Focus at this level is on understanding the rationale for completing the square as a viable method for solving quadratics as called out in A1.A.REI.B.3. As this is a modeling standard, students should be working with quadratic expressions embedded in real-world situations and let the context of the situation drive the structure of how the expression is written in order to answer a question embedded in the situation.

Additionally, the introduction of rational exponents and practice with the properties of exponents in high school further widens the field of operations students will be manipulating. As this is a modeling standard, the exponential expressions should be embedded in real-world situations. As with the quadratic expressions, this provides a context for seeing structure in the expression and allows students to see when and why it is beneficial to view them in different forms.

#### **Level 4:**

As students solidify their understanding and become more proficient with working with quadratics embedded in real-world situations, they should be able to not only identify if they need to factor a quadratic or complete the square with a quadratic, but also provide a justification based on the context of the problem to explain why they chose a specific form for the quadratic expression. Additionally, beyond simply finding the zeros of an expression and determining their meaning within a real-world context, students should develop an understanding of how many zeros are generated from quadratic expressions and what the number of zeros can mean in the context of a real-world situation.

As students deepen their understanding of maximum and minimum values for a quadratic embedded in a real-world situation, they should be able to explain the benefit of using completing the square as a method. Students at this level should also possess an understanding of the maximum or minimum values within context and be able to explain their relationship. Ultimately, students should be able to work with real-world situations that ask them to use the same expression and find and interpret intercepts, find and interpreting maximum or minimum values, interpret all values in terms of the context of the problem, and justify how the structure of the expression was used in order for the values to be determined in the first place.

Students should continue to demonstrate an understanding of seeing structure in expressions by not only being able to rewrite exponential expressions in various forms, but also in mathematically justifying the steps to reach the desired rewritten form and describing when and why the rewritten form would be beneficial. Students should encounter exponential expressions of increasing difficulty embedded in increasingly more complex real-world situational problems.

## ARITHMETIC with POLYNOMIALS and RATIONAL EXPRESSIONS (A.APR)

### Standard A1.A.APR.A.1 (Major Work of the Grade)

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract linear expressions with rational coefficients.	Add two simple polynomial expressions.  Multiply two binomial expressions.  Subtract two simple polynomial expressions.	Add polynomial expressions.  Subtract two polynomial expressions.  Multiply polynomial expressions.  Explain what it means for polynomials to be closed under the operations of addition, subtraction, and multiplication.	Add, subtract, and multiply multiple polynomial expressions, including situations involving more than one operation.  Explain the similarities that exist between adding, subtracting, and multiplying integers and adding, subtracting, and multiplying polynomials.

### Instructional Focus Statements

#### Level 3:

The development of an understanding of polynomials in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions like  $7+4$  and  $11$  as referring to different entities:  $7+4$  might be seen as a calculation and  $11$  as its answer. They come to understand that different expressions are different names for the same numbers, that properties of operations allow numbers to be written in different but equivalent forms, and that there are often benefits from writing numbers in various forms. They come to see numbers as forming a unified system: the number system. A similar evolution takes place in algebra. At first, algebraic expressions are simply numbers in which one or

more letters are used to stand for a number which is unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions as naming some underlying thing. Additionally, they reach the understanding that polynomials form a system in which they can be added, subtracted, and multiplied and that this system is closed for these operations. It is important that students understand this system beyond simply a rote set of steps which allows students to add, subtract, and multiply polynomials.

**Level 4:**

Students with a deep understanding of polynomials should be able to simplify complex expressions that involve multiple operations. Additionally, they will be able to explain in both verbal and written form the similarities that exist between calculating with integers and adding, subtracting, and multiplying polynomials.

**Standard A1.A.APR.B.2 (Supporting Standard)**

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**Scope and Clarifications:**

Graphing is limited to linear and quadratic polynomials.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Graph a linear equation.  Identify zeros of a linear equation.  Identify the zeros of a quadratic equation when a graph is provided.  Choose the zeros for a given quadratic equation in factored form when the quadratic has a lead coefficient of 1.	Choose a quadratic graph to represent a given equation.  Choose the zeros for a given quadratic equation.	Identify the zeros of a polynomial equation of degree 3 or greater when the factorization is provided.  Find the zeros of a quadratic equation and use them to graph the quadratic equation.	Generate multiple polynomial equations that would have this set of zeros, given a set of zeros,  Choose the zeros of a polynomial equation of degree 3 or greater when the factorization is not provided.  Explain what zeros of polynomials are, multiple ways to find them, and how zeroes are helpful when creating a graph.

**Instructional Focus Statements**

**Level 3:**

Polynomial functions are, on the one hand, very elementary, in that they are built up out of the basic operations of arithmetic. On the other hand, they turn out to be amazingly flexible, and can be used to approximate more advanced functions such as trigonometric and exponential functions in later courses. Experience with constructing polynomial functions satisfying given conditions is useful preparation not only for calculus, but for understanding the mathematics behind curve-fitting methods used in applications to statistics and computer graphics.

The first step in developing this understanding is to construct a rough graph for polynomial functions by using their zeros. Eventually, this progression will lead to constructing polynomial functions whose graphs pass through any specified set of points in the plane. It is important that students in this early stage make the connection between the graphical and algebraic representation of zeroes and that they are not simply following a rote procedure but provide evidence of an understanding of this connection.

**Level 4:**

At this level of understanding, students should be demonstrating strong command of the relationship that exists between an algebraic representation that elicits zeroes of a polynomial function and the graphical representation of zeros moving fluidly between the two. Additionally, they should be able to provide a mathematical explanation of the relationship between algebraic and graphical representations of zeros.

## CREATING EQUATIONS\* (A.CED)

### Standard A1.A.CED.A.1 (Major Work of the Grade)

Create equations and inequalities in one variable and use them to solve problems.

### Scope and Clarifications: (Modeling Standard)

Tasks are limited to linear, quadratic, or exponential equations with integer exponents.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose a linear equation in one variable that represents a simple, real-world situation.</p> <p>Solve a one variable linear equation.</p> <p>Solve a one variable linear inequality.</p> <p>Identify if a real-world situation can be represented by a linear, quadratic, or exponential equation.</p> <p>Determine if the solution to a real-world situation requires a one-variable or two variable equation or inequality.</p>	<p>Create and solve a one variable linear equation that represents a simple, real-world situation.</p> <p>Create and solve a one variable linear inequality that represents a simple, real-world situation.</p> <p>Choose a quadratic or exponential equation to represent a simple, real-world situation.</p> <p>Choose a quadratic or exponential inequality to represents a simple, real-world situation.</p> <p>Solve a simple one-variable quadratic equation.</p>	<p>Create and solve a one variable linear, quadratic, or exponential equation that represents a real-world situation.</p> <p>Create and solve a one-variable linear inequality that represents a real-world situation.</p> <p>Create and solve a one-variable quadratic or exponential inequality that represents a simple real-world situation.</p>	<p>Create a real-world situational problem to represent a given linear, quadratic, or exponential equation or inequality.</p> <p>Create and solve a one-variable quadratic or exponential inequality that represents a real-world situation.</p>

## **Instructional Focus Statements**

### **Level 3:**

In Algebra I, the variety of function types that students encounter allows students to create more complex equations and work within more complex situations than what has been previously experienced.

As this is a modeling standard, students need to encounter equations and inequalities that evolve from real-world situations. Students should be formulating equations and inequalities, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems. Real-world situations should elicit equations and inequalities from situations which are linear, quadratic, and exponential in nature. It is imperative that students have the opportunity to work with each of these function types equally.

### **Level 4:**

When given an equation or inequality, students can generate a real-world situation that could be solved by a provided equation or inequality demonstrating a deep understanding of the interplay that exists between the situation and the equation or inequality used to solve the problem.

Additionally, students should continue to encounter real-world problems that are increasingly more complex. Students should be using the modeling cycle to solve real-world problems.

**Standard A1.A.CED.A.2 (Major Work of the Grade)**

Create equations in two or more variables to represent relationships between quantities; graph equations with two variables on coordinate axes with labels and scales.

**Scope and Clarifications: (Modeling Standard)**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose a linear equation that represents a simple real-world or mathematical situation.</p> <p>Choose a linear graph to represent a simple real-world or mathematical situation.</p> <p>Determine if the solution to a real-world or mathematical situation requires a one-variable or two-variable equation.</p>	<p>Create and graph a two variable linear equation that represents a simple, real-world or mathematical situation.</p> <p>Choose a quadratic, exponential, absolute value, step, or piecewise equation to represent a simple mathematical situation.</p> <p>Choose a quadratic, exponential, absolute value, step, or piecewise graph to represent a simple mathematical situation.</p>	<p>Create and graph a two variable linear, quadratic, exponential, absolute value, step, or piecewise equation that represents a mathematical situation.</p>	<p>Create and graph a two-variable linear, quadratic, exponential, absolute value, step, or piecewise equation that represents a real-world situation.</p> <p>Create a real-world situational problem to represent a given linear, quadratic, exponential, absolute value, step, or piecewise equation.</p>

**Instructional Focus Statements**

**Level 3:**

In Algebra I, the variety of function types that students encounter allows students to create more complex equations and work within more complex situations than what has been previously experienced.

As this is a modeling standard, students need to encounter equations that evolve from both mathematical and real-world situations. Students should be formulating equations, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to mathematical and real-world problems.

Mathematical situations should elicit equations from situations which are linear, quadratic, exponential, absolute value, step, or piecewise in nature. It is imperative that students have the opportunity to work with each of these function types equally.

As there are no assessment limits to this standard, it is imperative that students be exposed to creating and graphing all function types students work with in the function domain.

**Level 4:**

One of the most natural situations for students to create an equation or graph from is a real-world situation. Students need to be exposed to variety of real world situations that illicit the wide variety of function types embedded within the Algebra I course. Students should encounter real-world problems that are increasingly more complex over time. They should be using the modeling cycle in order to develop and provide justification for their solutions.

Additionally, students should be posed with an equation and then asked to generate a real-world situation that could be solved by a provided equation. Students with this capability are demonstrating a deep understanding of the interplay that exists between the situation and the equation used to solve the problem.

**A1.A.CED.A.3 (Major Work of the Grade)**

Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

**Scope and Clarifications: (Modeling Standard)**

For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify unknown values in a real-world problem.</p> <p>Describe the difference of a viable solution and a non-viable solution.</p>	<p>Choose an equation or inequality that models the constraint on a variable given a contextual problem.</p> <p>Determine when a solution would be viable or non-viable, given an equation or inequality that represents a real-world problem.</p> <p>Determine the viability of each solution, given an equation or inequality that represents a contextual situation and a set of possible solutions.</p>	<p>Write an equation or inequality that models the constraint on a variable given a contextual problem.</p> <p>Write a system of equations or inequalities that models the constraint on a variable given a contextual problem.</p> <p>Explain constraints on a variable in context of a real-world problem and interpret solutions to determine the viability by using a graph, table, and equation.</p> <p>Justify solutions that model real-world problems when there are limitations on a variable.</p> <p>Interpret solutions as viable or</p>	<p>Create and provide a solution to a real-world problem that has natural limitations on variables. Explain the solution and its viability using multiple representations (i.e. table, graph, equation) and precise mathematical language.</p> <p>Explain examples of both viable and nonviable solutions in context of a real-world problem.</p> <p>Use multiple representations to justify a solution’s viability and explain when one representation elicits a more efficient justification.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		nonviable options in a modeling context using multiple representations (i.e. table, graph, equation).	

### **Instructional Focus Statements**

**Level 3:**

Students begin to develop a conceptual understanding of creating equations that represent contextual problems beginning as early as kindergarten. In the middle grades, students extend this understanding to linear equations. In Algebra I, students expand on that knowledge of variables and equations by identifying and quantifying any limitations present on the value of variable amounts within real-world problems prior to solving the problem. Through this modeling standard, students should explore the impact of constraints on variable amounts and see how the constraints effect the table, graph, and equation that represents the real-world problem. Students should also be able to define constraints and write an equation, inequality, or system of equations or inequalities that represents the constraint. Students often have a difficult time creating equations that represent constraints. They often can verbalize the constraint but have a difficult time translating it into a mathematical representation. Using multiple representations of the contextual situation can help students see how the constraint effects the problem. For example, students should have the opportunity to explore the impact of the constraint algebraically, graphically, and numerically.

Instruction should provide a variety of contexts with variable limitations and allow students to explain the solutions in context of the real-world problem. Students should experience both viable and non-viable solutions and make sense of them with respect to the contextual problem. Students should experience problems containing exclusively viable solutions, exclusively non-viable solutions, and problems that generate both viable and non-viable solutions. Students should experience problems where they have to decide the best way to report the solution. For example, when a student obtains a solution to a problem involving the amount of animals and the answer includes a fractional part, the student should make sense of the fractional part and determine the best way to report the amount of animals. Instruction should focus on students reporting solutions in context of the problem as this encourages students to make sense of and justify the viability of the solution. Since this is a modeling standard, students should have ample opportunity working with applications of equations and inequalities with real-world constraints (i.e., volume and linear programming).

**Level 4:**

Instruction at this level should provide opportunities for students to explore in-depth the impact of constraints and natural limitations on variables for a real-world problem. Students at this level of understanding should be given the opportunity to design their own real-world problems that would have

constraints on the unknown variables. Furthermore, students should be provided the opportunity to critique the solutions of others. Students need multiple opportunities to justify their thinking-by developing and providing logical arguments for the viability of solutions.

**Standard A1.A.CED.A.4 (Major Work of the Grade)**

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

**Scope and Clarifications: (Modeling Standard)**

- i) Tasks are limited to linear, quadratic, and exponential equations with integer exponents.
- ii) Tasks have a real-world context.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose equivalent forms of a given linear real-world formula.	Rearrange real-world linear formulas to highlight a quantity of interest.  Choose equivalent forms of a given quadratic real-world formula.  Choose equivalent forms of a given exponential real-world formula.	Rearrange real-world quadratic formulas to highlight a quantity of interest.  Rearrange real-world exponential formulas to highlight a quantity of interest.	Rearrange real-world linear, quadratic, and exponential formulas and explain the benefit of solving the formula for the various variables.

**Instructional Focus Statements**

**Level 3:**

In previous grades, students have focused on rearranging linear formulas to highlight a quantity of interest. In Algebra I, the linear formulas student work with should be fairly complex. Additionally, they should be working with quadratic and exponential equations with integer exponents. As this is a modeling standard, students should be encountering formulas that come from real-world situations.

Additionally, students need to be developing a conceptual understanding of why they might need to write formulas in different ways and what the benefit would be to these various representations of the same real-world formula.

**Level 4:**

Students need to be exposed to a wide variety of real-world formulas increasing in complexity over time. Additionally, it is imperative that they are able to explain why formulas might need to be expressed in different ways and the benefit that each form provides.

## REASONING with EQUATIONS and INEQUALITIES (A.REI)

### Standard A1.A.REI.A.1 (Major Work of the Grade)

Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

#### Scope and Clarifications:

Tasks are limited to linear, quadratic, and absolute value equations with integer exponents.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose the inverse operations used in solving the equation, given a linear, quadratic, or absolute value equation and a list of steps representing the solution path.</p> <p>Choose a possible next step to solve the equation, given a linear, quadratic, or absolute value equation and a partial list of steps representing the solution path.</p> <p>Arrange steps in the order they should be applied to solve an equation, given a linear, quadratic, or absolute value equation and a list of unordered steps representing the solution path.</p> <p>Explain the reasoning for each step, given a linear equation and a list of</p>	<p>Explain the reasoning for each step, given a quadratic equation and a list of steps representing the solution path.</p> <p>Explain the reasoning for each step, given a quadratic equation and a list of steps representing the solution path.</p> <p>Identify how it can be determined if no solution or infinitely many solutions exists, given an equation and a list of steps representing the solution path.</p>	<p>Solve linear, quadratic, and absolute value equations using multiple solution strategies and explain each step in the solution path.</p> <p>Construct a viable argument to justify a chosen solution path used to solve a linear, quadratic, and absolute value equation.</p> <p>Compare the steps in each and determine which solution path is most efficient, given an equation with multiple solution paths.</p> <p>Explain when an equation has no solution or infinitely many solutions.</p>	<p>Solve the problem, explain each step in the solution path, and justify the solution path chosen, given a real-world problem and an equation that represents the contextual situation.</p> <p>Compare and contrast two given solution paths to a contextual problem and construct a viable argument on which method is most efficient.</p> <p>Correct the mistakes in the solution path and provide an explanation of the misconception using precise mathematical vocabulary, given a list of steps representing an inaccurate solution for a linear, quadratic, and absolute value equation.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
steps representing the solution path.			

### Instructional Focus Statements

#### Level 3:

In Algebra I, students should develop a conceptual understanding of solving equations as a reasoning process to determine a solution that satisfies the equation rather than a procedural list of steps. Instruction should focus on students creating and determining solution paths or each unique equation and providing a viable argument to justify the chosen solution path. Students should also be able to explain how, when, and why equations have no solution or infinitely many solutions. To help give meaning to these solution types, discussion should focus on the solution being a value of the variable that makes the equation true. This will help students make the connections that an equation has no solution because there is no value that can maintain equivalency and an equation has infinitely many solutions because all values used for the variable create a true equivalency statement.

Students should understand that a problem can have multiple entry points and instruction should be focused on solving equations using a reasoning process of centered around inverse operations and order of operations. Students develop a conceptual understanding of operations in previous grades and they should deepen their understanding of the interplay that exist between the operations. To illustrate maintaining equivalency, a visual and/or concrete model of a balance scale can be used to aid students in understanding that the same inverse operations are being applied to the whole left side and the whole right side of an equation. Emphasizing equivalency is vital in developing a conceptual understanding of solving equations and preventing common misconceptions.

Students should understand that the solution path they choose to solve any equation must result in a series of equivalent equations all of which have the same solution set. As students apply inverse operations to solve equations, they should be able to explain why equality holds true when performing the selected operation to both sides of the equation. In this course, students should be exposed to linear, quadratic, and absolute value equations, as other function types will be explored in Algebra II.

#### Level 4:

As students develop a deeper understanding of solving equations and explaining their solution methods, instruction can be integrated with the application in contextual situation. Students should be able to construct equations that represent a contextual situation as well as create contextual situations to represent a given equation. As students develop a deep understanding of the relationship that exists between the type of function and the context, they can be given functions embedded in real-world situations. When they are given a contextual situation and an equation, students should be able to

determine what each part of the equation represents as it relates to the context. They should also be able to solve the equation and create a viable argument to justify their solution path. Students should understand that there are various ways to solve problems and justifying their steps will help them solidify their understanding of solving equations as well as the most efficient solution path. This standards pairs nicely with A1.A.CED.A.1, as it supports the idea of making connections between an equation and its context.

To challenge students to follow a thought process other than their own, they can be asked to critique or correct the solution paths of others. Students will develop a deeper level of understanding if they are given solution paths with incorrect steps in the process or invalid justifications and asked to correct the process or write justifications and defend them.

**Standard A1.A.REI.B.2 (Major Work of the Grade)**

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**Scope and Clarification:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Determine if a linear equation in one variable has one solution, infinitely many solutions, or no solutions.</p> <p>Solve linear equations in the form <math>x + p = q</math> and <math>px = q</math>.</p>	<p>Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</p> <p>Solve linear inequalities in the form <math>px + q &gt; r</math> or <math>px + q &lt; r</math>.</p>	<p>Solve linear equations in one variable with coefficients represented by letters.</p> <p>Solve multi-step linear equations.</p> <p>Solve multi-step linear inequalities in one variable.</p>	<p>Create a linear equation in one variable that has infinitely many solutions and provide mathematical justification as to why.</p> <p>Create a linear equation in one variable that has one solution and provide mathematical justification as to why.</p> <p>Create a linear equation in one variable that has no solution and provide mathematical justification as to why.</p> <p>Explain the difference between a linear equation and a linear inequality and provide examples when each would be used.</p>

## **Instructional Focus Statements**

### **Level 3:**

It is important that students begin with the understanding developed in A1.A.REI.A.1 that solving any equation is a process. With this understanding, students can organize the various techniques for solving equations into a coherent picture instead of viewing each part in isolation.

For example, solving linear equations involves only steps that are reversible (adding a constant to both sides, multiplying both sides by a non-zero constant, transforming an expression on one side into an equivalent expression). Thus, students can deduce that solving linear equations does not produce extraneous solutions.

As linear equations and inequalities have been addressed in previous grades, this is the opportunity for students to interact with more complex equations and really practice looking at the big picture for solving linear equations and inequalities. Additionally, students need to be developing a conceptual understanding of why they might need to write formulas in different ways and what the benefit would be to these various representations of the same real-world formula.

### **Level 4:**

Students need to be pushed to really solidify their understanding of the many ways to solve quadratic equations supply mathematical justification to the decisions they make. At this level, students can not only solve quadratic equations, but they can articulate the mathematical underpinnings of the various strategies.

Additionally, they are able to connect the various strategies. For example, are the quadratic formula and completing the square related? If so, describe how. If not, explain why.

**Standard A1.A.REI.B.3 (Major Work of the Grade)**

Solve quadratic equations and inequalities in one variable.

**A1.A.REI.B.3a** Use the method of completing the square to rewrite any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

**A1.A.REI.B.3b** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions.

**Scope and Clarifications:**

Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions. Note: solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials. This is formally assessed in Algebra II.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose the correct factorization when given a quadratic expression where <math>a = 1</math>.</p> <p>Factor a perfect square trinomial, given an expression.</p> <p>Choose the solution to a simple one variable quadratic equation.</p> <p>Determine if a given test point is a solution, given a segmented number line that represents the intervals for a quadratic inequality in one variable.</p> <p>Algebraically determine if a given value is a solution for a quadratic inequality in one variable.</p>	<p>For an expression written in the form of <math>ax^2 + bx + c</math>, find the value of <math>c</math> that makes the expression a perfect square trinomial and then rewrite the expression as the square of the binomial.</p> <p>Use coefficients from the standard form of a quadratic equation appropriately in the quadratic formula.</p> <p>Solve a simple one-variable quadratic equation.</p>	<p>Use the method of completing the square to rewrite a quadratic equation when <math>a = 1</math>, in the form of <math>(x - p)^2 = q</math>.</p> <p>Use the method of completing the square to rewrite a quadratic equation when <math>a \neq 1</math>, in the form of <math>(x - p)^2 = q</math>.</p> <p>Derive the quadratic formula from standard form using the method of completing the square and explain the steps.</p> <p>Solve quadratic equations in one variable using multiple strategies.</p>	<p>Develop a logical argument justifying why completing the square would be the most efficient way to solve a quadratic equation.</p> <p>Use a given method to complete the problem and justify the steps in the solution path, when given a problem partially solved.</p> <p>Solve one-variable quadratic equations, identify the strategy chosen, and explain why the chosen strategy best suits the initial form of quadratic equation.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Choose the possible intervals that could be solutions to a quadratic inequality in one variable.	Determine if a quadratic equation in one-variable has real solutions or complex solutions.  Solve a simple quadratic inequality when $a = 1$ in one variable.	

### Instructional Focus Statements

#### Level 3:

This standard builds on the foundational understanding of the structure of quadratic equations. In the standard A1.A.SSE.B.3, students developed a concrete understanding of completing the square to produce equivalent forms of a quadratic expression. More detailed instructional guidance for completing the square can be found within the A1.A.SSE.B.3 instructional focus document. This guidance hinges on understanding this concept through direct modeling strategies such as using models and algebra tiles, including the progression of learning to more abstract strategies such as using algorithms. For A1.A.REI.B.3, students should understand how to rewrite quadratic equations written in standard form into the form  $(x - p)^2 = q$  by completing the square. Student's attention should be focused on rewriting the equation for the purpose of deriving the quadratic formula. In addition, connections should be made between completing the square and vertex form of a quadratic function which can be used in A1.F.IF.C.6 when students graph functions and identify the maxima or minima. To build conceptual understanding, students should explain the steps required to rewrite an equation in vertex form and derive the quadratic formula through the process of completing the square.

It is important that, just like for linear equations, students begin with the understanding developed in A1.A.REI.A.1 that solving any equation is a process. Students should interact with the multiple strategies for working with quadratic equations discovering and internalizing when each strategy is most beneficial. The strategies should not live in isolation but as a part of a much larger conversation around the merits of each.

In previous grades and course work, students developed an understanding of linear inequalities utilizing both a number line for one variable inequalities and the coordinate plane for two variable inequalities. Students should build on both their prior knowledge of linear inequalities and their developing understanding of quadratic equations to begin developing a conceptual understanding of how to solve quadratic inequalities in one variable, what the solutions to quadratic inequalities look like, and why they look that way. As students develop a conceptual understanding of the various strategies used to solve quadratic equations, they use these same strategies as they are exploring the boundaries of solution intervals for quadratic inequalities. Instruction should focus on student exploration of solutions for simple quadratic inequalities in one variable where they discover the points

that segment a number line into regions and connect that back to the quadratic inequality. Students should test points along the continuum of the number line first to discover the boundaries and second to determine which intervals hold viable solutions for the inequality. In culmination, students develop an understanding that the solutions to a quadratic inequality are an infinite number of point within intervals. All in all, the focus for Algebra I is for students to develop a strong conceptual understanding of simply quadratic inequalities so that in future course work when more complex quadratic inequalities or quadratic inequalities embedded in real-world problems are encountered, they are prepared to think through how to solve them and interpret the meaning of the solution.

Additionally, it is important to note that students are not required to find complex roots, but they are required to have an understanding of when they will not get a real solution.

**Level 4:**

Students should solidify their understanding of the many ways to solve quadratic equations by supplying mathematical justification to the decisions they make. At this level, students can not only solve quadratic equations, but they should be able to articulate the mathematical underpinnings of the various strategies. Instruction should focus on students not only being able to explain and justify their reasoning for choosing to solve a quadratic equation by completing the square, but also explaining the step by step process for solving a quadratic equation by completing the square. Additionally, students should be able to connect the various strategies in solving quadratic equations. For example, students should explore and be able to explain the similarities and differences between the quadratic formula and completing the square

To solidify the understanding of solving quadratic inequalities, student should understand that a perfect square binomial and a trinomial result in segmenting the number line into three and two regions respectively when determining test intervals for solution sets. Students should also be able to explain why an infinite set of solutions bounded within an interval is the solution to a quadratic inequality.

**Standard A1.A.REI.C.4 (Supporting Content)**

Write and solve a system of linear equations in context.

**Scope and Clarifications:**

Solve systems both algebraically and graphically. Systems are limited to at most two equations in two variables.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify the solution to a system of linear equations in two variables given a graphical representation of the system.</p> <p>Identify the solution to a system of linear equations in two variables given a tabular representation of the system.</p> <p>Use substitution to determine if a given solution satisfies a given system of linear equations.</p>	<p>Solve a system of linear equations by graphing.</p> <p>Determine if two linear equations create parallel lines, the same line, or intersecting lines.</p>	<p>Solve a system of linear equations in two variables algebraically using the substitution method.</p> <p>Solve a system of linear equations in two variables algebraically using the elimination method.</p> <p>Write a system of linear equations in two variables given a real-world context.</p> <p>Interpret the solution of a system of linear equations in two variables in relationship to a context.</p> <p>Justify why a system of linear equations in two variables may have one solution, no solutions, or infinitely many solutions.</p>	<p>Create a real-world scenario to represent a system of linear equations in two variables.</p> <p>Determine if the solution to a system of linear equations in two variables is reasonable in relationship to a context.</p> <p>Justify whether the substitution or elimination method would be more efficient to solve a system of linear equations in two variables.</p>

## **Instructional Focus Statements**

### **Level 3:**

Students should understand that the solution to a system of linear equations is the point, ordered pair, at which two linear graphs intersect. Instruction should include the use of multiple representations consisting of graphs and tables to help students conceptualize the solution and support their ability to solve a system of linear equations in two variables algebraically. When focusing on the graphical representation, students should be able to approximate the intersection of the graphs created by hand and with technology. When creating a graph by hand, students should begin with solutions that lie on integer points on the graph and move on to solutions that do not lie on integer points and discuss approximate solutions that could exist. The calculation of the intersection on a graphing calculator may be used to allow students to continue this exploration of approximate points of intersection. Additionally, students should solidify their solutions by substituting their approximated solutions into the system of linear equations in two variables, which will foster the importance of accuracy in the solutions.

Algebraically, students should have opportunities to utilize multiple methods of solving a system of linear equations in two variables. Students should be introduced to the substitution method through a conversation about the property of substitution and how to substitute a value of a variable in place of the variable. Building on this idea can help solidify the understanding of solving for a variable and then substituting the equivalent expression in for that variable in the other equation. The elimination method should be introduced based on the concept that we cannot solve for more than one unknown value at a time. Instruction should begin with a system of linear equations where one variable has opposite values for the coefficients and results in equaling zero when added together. This will allow students to understand how to "eliminate" one variable in order to solve for the other variable. Students should then engage in problems where a variable cannot be initially eliminated and make use of rewriting the equations in equivalent forms that will lend to one variable having opposite value coefficients by using the multiplication property of equality. Emphasis needs to be placed on the importance of finding both variables since the solution is an ordered pair which correlates to the point where the two equations intersect on the graph. Students should solidify their understanding that the solution is the value of the variables that satisfies both equations. In Algebra II, students will expand their work to include a system of three equations with three variables as well as a system containing a linear and a quadratic function.

Students should have opportunities to engage in real-world problems in which they must determine if an approximation of a solution using a graph or an exact solution using other methods is most appropriate for the problem. Students should differentiate among problems where there is one solution, no solutions, or infinitely many solutions and justify the meaning for these results. Additionally, students should understand that one solution is represented by two graphs that intersect at one point, no solution is represented by two graphs that are parallel and never intersect, and many solutions is represented by two graphs that lie on the same line and are infinitely intersecting.

### **Level 4:**

Given a system of linear equations in two variables, students should be able to write a real-world scenario to represent the system and construct a graphical model either by hand or using technology. Students should also be able to determine the reasonableness of the solution and then justify their

thinking. Instruction should also allow students to demonstrate a deeper level of understanding by choosing the most appropriate algebraic or graphical method for solving a system of linear equations. Students should be expected to justify their reasoning and explain the steps in their chosen solution path.

**Standard A1.A.REI.D.5 (Major Work of the Grade)**

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Plot the ordered pairs on a coordinate plane, given a table of values.</p> <p>Determine if points on a graph form a line.</p> <p>Represent the set as a table of values, given a set of ordered pairs.</p> <p>Use technology to produce the graph of a given equation.</p>	<p>Find corresponding values of <math>y</math>, given an equation and a set of values of <math>x</math>.</p> <p>Graph ordered pairs to identify whether they represent a linear or non-linear function.</p>	<p>Find a set of solutions that can be used to create the graph, given an equation.</p> <p>Interpret the graph of an equation as the solution set to the equation with two variables.</p> <p>Explain why the points on a curve (or line) would be continuous.</p> <p>Explain the relationship between the graphical representation and the solutions (ordered pairs) to the equation, given a real-world situation.</p>	<p>Analyze the pattern of the points to determine the type of function represented by the graph, given a set of ordered pairs.</p> <p>Determine if the graphical representation for a real-world situation is continuous or discrete and justify the reasoning.</p> <p>Use the regression feature on a graphing calculator to determine the curve of best fit given a real-world problem.</p>

**Instructional Focus Statements**

**Level 3:**

As students increase their understanding of graphs, they should begin to make the connection between the solutions of an equation and its graphical representation. Students have some understanding of graphs of linear equations from grade 8 and build on that knowledge to include quadratic and

other non-linear equations. It is imperative that instruction focus on the multiple representations of an equation and make connections between the equation, the table, and the graph throughout the learning process. From an equation, students should be able to find the ordered pairs that are solutions to the equation and generate a table and a graph representing those solutions. Allowing students to choose their own input values can create a variety of different tables that all represent the same equation. This can foster classroom discourse to help students to develop the understanding of the infinite amount of points that lie on a line or curve. Often times, students interpret solutions at only integer values. Instruction should provide opportunities for discussion of all solution sets and ways to represent these values on the coordinate plane. As students evaluate graphs, they should be able to determine if all possible solutions are represented to discover the meaning of a continuous line or curve.

As students solidify their understanding of graphical representations, they should extend their learning to include real-world problems. When solutions are graphed, they should be able to determine if the graphical representation of the real-world problem is continuous or discrete and justify their reasoning. In addition, students should be able to identify the domain and range of the function based on their multiple representations.

#### **Level 4:**

Instruction can extend students learning by making connections to real-world problems. When given a table that represents real-world data, discussion could be extended beyond possible solutions to solutions that fit the context. Students should have experience with recognizing limits of an equation within context and making connections to the context effecting whether solutions would be continuous or discrete. For example, if the original input represents time, it should be understood that time cannot be negative thus the equation could not be used for negative x-values.

This standard can also be connected with A1.S.ID.B. 2 by having students use the regression feature on a graphing calculator to determine the curve of best fit. As students experiment with multiple function types, they should be able to determine which regression equation will produce the curve of best fit for a given real-world situation and justify their reasoning. In addition, equations in appropriate context can be used for making predictions.

**Standard A1.A.REI.D.6 (Major Work of the Grade)**

Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the approximate solutions using technology.

**Scope and Clarifications: (Modeling Standard)**

Include cases where  $f(x)$  and/or  $g(x)$  are linear, quadratic, absolute value, and exponential functions. For example,  $f(x) = 3x + 5$  and  $g(x) = x^2 + 1$ . Exponential functions are limited to domains in the integers.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the solution of the equation $f(x) = g(x)$ , given two linear equations $f(x)$ and $g(x)$ .	Identify the solution(s) for $f(x) = g(x)$ when $f(x)$ and $g(x)$ are linear, quadratic, absolute value or exponential, given graphs of two equations $f(x)$ and $g(x)$ .  Choose the solution(s) for $f(x) = g(x)$ when $f(x)$ and $g(x)$ are linear, quadratic, absolute value or exponential, given two equations $f(x)$ and $g(x)$ .	Approximate the solution(s) for $f(x) = g(x)$ using technology when $f(x)$ and $g(x)$ are linear, quadratic, absolute value or exponential, given two equations $f(x)$ and $g(x)$ embedded in a real-world situation.  Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ .	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and explain the meaning of the solution in terms of a real-world context.

**Instructional Focus Statements**

**Level 3:**

In developing an understanding of what it means to find the solution to two equations using graphing, it is very important that just as we did not want algebraically solving equations to become a series of steps unsupported by reasoning, we want to make sure that graphically solving them the reasoning piece is not left out either. The simple idea that an equation can be solved (approximately) by graphing can often lead to a rote series of steps involving simply finding the intersection point(s) without employing the reasoning of what is actually occurring. Explaining why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$  involves a rather sophisticated series of thinking as students must connect the idea of two equations in two variables and how that relates to a single equation in one variable and then understand how

both connect to a point(s) on a coordinate plane which is built around two variables. Thus, it is imperative that students reason through this process without being given a truncated set of meaningless steps to follow.

As this is a modeling standard, Students should be formulating equations, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions built out of real-world situations.

In Algebra I, students are focusing on linear, quadratic, absolute value, and exponential functions. Students need the opportunity to interact with all of these function types. Additionally, they need to encounter situations where  $f(x)$  and  $g(x)$  are different function types. These should increase in difficulty over time.

Additionally, it is important to note that students are not required to find complex roots, but they are required to have an understanding of when they will not get a real solution.

**Level 4:**

Students should continue to be exposed to a wide variety of linear, quadratic, absolute value, and exponential functions with increasing difficulty embedded in real-world situations. Additionally, they need to explain the meaning of the solution in terms of the real-world context.

**Standard A1.A.REI.D.7 (Major Work of the Grade)**

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

<b>Students with a level 1 understanding of this standard will most likely be able to:</b>	<b>Students with a level 2 understanding of this standard will most likely be able to:</b>	<b>Students with a level 3 understanding of this standard will most likely be able to:</b>	<b>Students with a level 4 understanding of this standard will most likely be able to:</b>
Identify if a point is a solution to a linear inequality in two variables.  Identify if a point is a solution to a system of linear equalities in two variables.	Choose the graphical representation of the solution to a linear inequality in two variables.  Choose the graphical representation of the solution set to a system of linear inequalities in two variables.	Graph the solutions to a linear inequality in two variables as a half-plane.  Graph the solution set to a system of two linear inequalities in two variables as the intersection of the corresponding half-planes.	Graph the solution set to a system of more than two linear inequalities in two variables.

**Instructional Focus Statements**

**Level 3:**

Instruction should focus on extending a student’s understanding of graphing linear inequalities in one variable on a number line to graphing linear inequalities in two variables on a coordinate plane. Students need to make the connection as to why inequalities have multiple solutions. It is important that students understand why they are shading not simply following a set of steps without conceptual understanding.

**Level 4:**

As students develop a strong command of systems of linear inequalities in two variables, they need to experience a wide variety of systems increasing in difficulty including those with more than two inequalities including those having no solution.

## INTERPRETING FUNCTIONS (F.IF)

### Standard A1.F.IF.A.1 (Major Work of the Grade)

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

#### Scope and Clarification:

There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Determine if a given table of values represents a function.</p> <p>Identify the domain is the set of input values.</p> <p>Identify the range is the set of output values.</p>	<p>Determine if a given graph represents a function.</p> <p>Find <math>f(a)</math> where <math>a</math> is a real number when given a function <math>f</math>.</p>	<p>Create an example of a function using a set of ordered pairs, a graph, and a table of values to show the correspondence between one input value (domain) and one output value (range).</p> <p>Explain the meaning of a function using correct mathematical vocabulary.</p>	<p>Identify the domain and the range and determine if the relationship represents a function, given a real-world situation.</p> <p>Determine if the domain and range is continuous or discrete and explain your reasoning, given a real-world situation.</p> <p>Create a real-world situation that does not represent a function and explain the reasoning.</p>

### Instructional Focus Statements

#### Level 3:

In grade 8, students used informal notation for functions and referred to the values used in those functions as input and output values. In Algebra I, students build on that understanding and begin using function notation and the mathematical language that describes a function. Students should make

the connection that input and output correlate respectively with the mathematical terms of domain and range. Students should understand that a function is a special relationship that assigns one input value (domain) to exactly one output value (range) in a pair of elements. Students should also recognize that in a function each element of the domain is unique, however elements of the range may repeat themselves. A common misconception is that students often interpret a function as each value of the range can only be paired with one domain value. It is important that students have multiple opportunities to explore functions and their ordered pairs as well as their graphs to understand a function.

Students should be able to determine if a relation is a function by examining sets of ordered pairs, graphs, or tables of values. As students begin to examine graphs to determine if they represent functions, using the vertical line test may be helpful. This does not provide a robust definition of a function that transfers well into other situations in future courses, but provides a concrete tool for students to use as they begin their study of graphs of functions. As students further develop their understanding of functions, they should understand that  $f(x) = y$  and can construct a viable argument using correct vocabulary to explain the meaning of a function. This standard can be integrated with A1.F.IF.A.2 which includes function notation in a real-world context.

#### **Level 4:**

Students should solidify their understanding by understanding that the domain of a function represents the input values and the range represents the output values. They should be able to identify the domain and the range in a real-world situation and understand that each element in the domain is unique and is paired with an element in the range. Students should construct a viable argument explaining why real-world situations can be represented as a function. As students solidify their understanding of what kind of real-world situations create functions, they should begin to describe real-world situations that are not functions and explain their reasoning.

**Standard A1.F.IF.A.2 (Major Work of the Grade)**

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Determine the input-output table that represents a given equation.</p> <p>Write an ordered pair given the function notation (e.g; <math>f(2) = 10</math> is the coordinate (2, 10).</p>	<p>Substitute input values into a given equation to reveal output values.</p> <p>Construct an input-output table given a set of ordered pairs.</p>	<p>Given a function that represents a real-world problem, determine what each variable represents.</p> <p>Given a function that represents a real-world problem, interpret the meaning of output values when given input values and vice versa.</p> <p>Use multiple representations to model a function in a real-world situation.</p>	<p>Explain situations when it is imperative to use function notation.</p> <p>Construct a viable argument to explain the solution of a function in a real-world situation.</p> <p>Given a real-world situation, identify and explain possible restrictions on the domain and range.</p>

**Instructional Focus Statements**

**Level 3:**

As students begin to use function notation, they should understand that  $f(x)$  is another name for  $y$  at a given  $x$  and is read as “the value of  $f$  at  $x$ ” or “ $f$  of  $x$ ”. Students should also understand that a variety of letters can be used for function notation such as  $g(x)$ ,  $h(x)$  or  $d(t)$ . Instruction can begin by giving students values of  $x$  and an equation written in function notation. Students should use substitution to determine the value of the function at the given  $x$  value. For example, when given  $f(x) = 5x - 8$ , students should be able to find  $f(-4)$  by substituting  $(-4)$  into the equation. This is helpful to build students’ understanding that function notation is not an arithmetic operation. Graphing would be a natural progression to help students understand that a table of values can be constructed from an equation written in function notation form and a graph can then be produced from the table. In addition, students should be given a function embedded in a real-world situation where they are to explain what the variables represent and what a solution means in the

given context. For example, in a contextual problem where a consultant earns a flat fee of \$25 plus \$40 per hour for a contracted job, students should be able to determine how much she will earn in 16 hours. Students should also be able to write the function in function notation, construct a table of values, graph the function, and determine the meaning of the key features of the graph in context. When a question related to the problem is asked, students should be able to solve for the numerical answer and provide an explanation as to what the answer means and why it is or is not viable. Instruction should also focus identifying the domain and the range that is represented in the situation and the reasoning why this is correct for the context of the problem. This standard pairs nicely with A1.F.IF.A.1.

**Level 4:**

With a deeper understanding, students can use functions and function notation in a real-world context. An example of this might be using the function  $h(t) = -16t^2 + 32t + 48$  where  $h$  represents the height of a baseball and  $t$  represents time. When the ball is thrown, students can determine how long it takes for the baseball to reach its maximum height or how long it takes for the ball to hit the field. Real-world problems such as this should be represented in multiple ways. Students should be able to write the function in function notation, construct a table of values, graph the function, and determine the meaning of the key features of the graph in context. When a question related to the problem is asked, students solve for the numerical answer and provide an explanation as to what the answer means and why it is or is not viable. Students should also be able to identify the domain and the range that is represented in the situation and explain why this is correct for the context of the problem. Additionally, students can apply their knowledge of functions to a real-world problem such as showing trends across time with a scatter plot, and show functional reasoning between two sets of information such as a name and social security or a name and a cell phone number.

**Standard A1.F.IF.B.3 (Major Work of the Grade)**

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**Scope and Clarifications: (Modeling Standard)**

Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

- i) Tasks have a real-world context.
- ii) Tasks are limited to linear functions, quadratic functions, absolute value functions, and exponential functions with domains in the integers.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify intercepts, maximums and minimums when provided a graphical representation of the function.</p>	<p>Identify intervals where a given function is increasing, decreasing, positive or negative when provided a graphical representation of the function.</p> <p>Identify key features of the graph or table of values, and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a linear function embedded in a real-world context.</p> <p>Identify evident intercepts, maximums and minimums when provided a table of values representing a linear, quadratic, or absolute function.</p>	<p>Identify all evident intercepts, maximums and minimums when provided a table of values representing an exponential function with domain in the integers.</p> <p>Identify all evident key features when provided a table of values representing a linear, quadratic, or absolute value equation.</p> <p>Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a quadratic function embedded in a real-world context.</p>	<p>Identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given a quadratic function embedded in a real-world context, graph the function.</p> <p>Identify key features of the graph, and interpret the meaning of the key features in relationship to the context of the problem, given an absolute value function embedded in a real-world context, graph the function.</p> <p>Create a real-world context that would generate a function with the provided attributes, given key</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	<p>Sketch a graph of the function, given a verbal description of the key features of a linear function.</p>	<p>Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing an absolute value function embedded in a real-world context.</p> <p>Identify key features of the graph and interpret the meaning of the key features in relationship to the context of the problem, given a graph of an exponential function with domain in the integers embedded in a real-world context.</p> <p>Sketch a graph of the function, given a verbal description of the key features of a quadratic or absolute value function.</p>	<p>features of a linear, quadratic, or absolute value function.</p>

### Instructional Focus Statements

**Level 3:**

Functions are often described and understood in terms of their key features and behaviors. Instruction for this standard should in part focus on helping students develop an understanding of how to identify key features and behaviors from both graphs and tables. That said, instruction should extend beyond simple identification from isolated graphs and tables. As this is a modeling standard, students need opportunities to develop an understanding of the relationship between key features/behaviors and the real-world situation that the function models. The focus should be on developing an understanding of what key features/behaviors are while also developing a strong understanding of their relationship and meaning to real-world situations.

Additionally, instruction should provide students with an opportunity to develop an understanding of not only how to identify key features/behavior in graphs and tables, but also on how to generate a graph when provided the key features/behaviors.

Instruction can be paired with standard A1.F.IF.C.6 where students generate linear, absolute value, and quadratic graphs from real-world situations. This pairing allows students the opportunity to generate a graph from a real-world situation, identify key features/behaviors, and then discuss their meaning as related to the real-world situation. That said, it is not a requirement of this course that students generate graphs of exponential equations with domain in the integers. Thus, discussions around this particular function family will need to be carefully planned out.

**Level 4:**

As students develop a deep understanding of this standard, they should be exposed to increasingly more complex real-world situations. Students should begin to create their own real-world scenarios that generate functions with a pre-determined list of key features/behaviors. Additionally, students with a deep understanding of this standard can interpret key features/behaviors from non-traditional linear, absolute value, quadratic, and exponential functions embedded in real-world situations.

**A1.F.IF.B.4 (Major Work of the Grade)**

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

**Scope and Clarifications: (Modeling Standard)**

For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function. There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Explain the difference between domain and range.</p> <p>Identify the domain and range from a table of values.</p> <p>Identify the domain and range from a discrete graph.</p> <p>Identify the domain and range, given a mapping.</p> <p>Identify the domain and range from a set of ordered pairs.</p> <p>Explain the difference between a continuous function and a discrete function.</p>	<p>Explain how domain relates to the definition of a function.</p> <p>Identify the domain, given the continuous graph of the function.</p> <p>Explain why a function is continuous or discrete given its graph.</p>	<p>Explain how the domain relates to the graph of a function.</p> <p>Explain why a function is continuous or discrete given an equation.</p> <p>Describe how a function's domain is affected when situated within a context.</p> <p>Explain if a function is continuous or discrete, given a context.</p>	<p>Determine an appropriate domain and range, given a function in context.</p> <p>Create a contextual situation to describe a function with a given domain and range.</p> <p>Using the definition of discrete and continuous, compare and contrast sequences and the functions used to model them.</p>

## **Instructional Focus Statements**

### **Level 3:**

As students begin their work with this modeling standard, instruction should start by making sure the students have a good understanding of domain and range. Providing tables, graphs, ordered pairs and mappings for students to identify the domain and range will lead to rich classroom discussion and the multiple representations will give students a deep understanding of the definitions of domain and range. Students should be able to explain how and why they have identified the domain and range in these varied forms. After students have a clear understanding of how to identify the domain and range, they should have experience relating the domain to the definition of a function. That is, every input must correspond to exactly one output value. They should also identify the domain given the graph of a function. Discussion should include comparing and contrasting discrete and continuous functions and identifying them from a graph and real-world situations.

An example that could be used to help students apply this standard might include small packs of skittles in the check-out line at a convenient store. If one bag contains 10 skittles and 2 bags contain 20 skittles, students can create a table of values and then determine if the graph is continuous or discrete. In this particular case, they should relate the number of packs to the domain and the number of skittles to the range. Students may ask if there has to be at least 1 bag at the check-out line and/or is there a limit of bags that could be placed there. After students have determined whether the domain is continuous or discrete, they should graph the function. In addition, careful attention must be paid to real-world problems where the domain might be continuous, but also restricted because of the context. An example might be a ball's height when thrown,  $h(t)$ , with respect to time. The domain would be continuous, but given the context,  $t$  must be greater than or equal to 0. As students solidify their understanding of different function families, they should begin to realize that the domain and range do not both have to be continuous or discrete. This is a good place to introduce step functions and explain how the domain might be is continuous while the range is discrete.

### **Level 4:**

As students are exposed to a variety of real-world problems, they should begin to realize how unique and different every problem can be, but that every modeling situation will have a domain and range. At a level 4 understanding, students should be able to determine the domain and range of real-world problems and have practice with a variety of scenarios. They should also have practice with identifying whether the function is continuous or discrete and explain why. As they develop a good understanding of domain and range, opportunities should be provided for students to create their own scenarios when given a domain and range. In addition, students should construct an argument explaining why their scenario represents the given domain and range. Instruction should also focus on connecting arithmetic and geometric sequences with the functions that model them. Students should justify why these sequences are discrete, while linear and exponential functions are continuous.

Examples which might be used to support understanding of this standard would include a scenario representing the cost of movie tickets per person. This is a discrete function and the domain and range are restricted to positive integer values. A continuous example might be a punter kicking a football over time ( $x$ ) compared to height ( $y$ ). This example also restricts the domain and range to positive values. As students understand how to determine the domain

and range and graphing functions, they should also be able to create their own scenarios with both discrete domain and range and continuous domain and range.

**Standard A1.F.IF.B.5 (Major Work of the Grade)**

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Scope and Clarifications: (Modeling Standard)**

- i) Tasks have a real-world context.
- ii) Tasks are limited to linear functions, quadratic functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

**Evidence of Learning Statements**

<b>Students with a level 1 understanding of this standard will most likely be able to:</b>	<b>Students with a level 2 understanding of this standard will most likely be able to:</b>	<b>Students with a level 3 understanding of this standard will most likely be able to:</b>	<b>Students with a level 4 understanding of this standard will most likely be able to:</b>
<p>Choose the average rate of change for a linear function when given a symbolic representation, table, or graph.</p> <p>Choose the estimated rate of change when given a graph of a linear function.</p>	<p>Calculate the average rate of change of a linear function when given a graph.</p> <p>Interpret the rate of change for a linear function in terms of a real-world context.</p> <p>Choose the estimated rate of change for a specific interval when given a quadratic function.</p>	<p>Calculate average rate of change when given an equation or table of a quadratic, absolute value, piece-wise, or exponential functions, where exponential functions are limited to domains in the integers.</p> <p>Interpret the average rate of change of a quadratic, absolute value, piece-wise, or exponential functions, where exponential functions are limited to domains in the integers.</p> <p>Estimate the average rate of change for a specific interval of a quadratic, absolute value, piece-wise, or exponential function when given a graph, where exponential functions are limited to domains in the integers.</p>	<p>Identify the average rate of change for specific intervals of a function as being greater or less than other intervals of the same function.</p> <p>Compare the average rate of change of multiple intervals of the same function and make connections to the real-world situation.</p> <p>Create a contextual situation and identify and interpret the average rate of change with a specific interval.</p>

## Instructional Focus Statements

### Level 3:

In grades 6 and 7, students began developing the understanding of ratios and proportional relationships. Their understanding of rate of change involved both ratios and proportions using similar triangles to show the additive and multiplicative conceptual underpinnings of the concept. In grade 8, students extended this understanding to functions by examining rate of change in linear functions. In high school, students should solidify this understanding for linear functions and generalize this concept to applying to additional function types. Students should make the connection that the rate of change is the ratio of the change between the dependent and independent variable. For linear functions, students have discovered that this ratio of change is constant between any two points on the line. Students should now make the connection that, for non-linear functions, the ratio of change is not constant due to the functions curvature. This results in the ability to calculate the average rate of change over a specified interval. For example, for the quadratic function  $f(x) = x^2$ , the average rate of change from  $x = 1$  to  $x = 4$  is  $\frac{f(4)-f(1)}{4-1} = \frac{16-1}{4-1} = \frac{15}{3} = 5$ . This is the slope of the line from  $(1, 1)$  to  $(4, 16)$  on the graph  $f$ . If  $f$  is interpreted as the area of a square of side  $x$ , then this calculation means that over this interval the area changes, on average, 5 square units for each unit increase in the side length of the square.

It is imperative that students gain a conceptual understanding of the average rate of change for a specified interval for non-linear functions. To grasp this idea, students should draw illustrations of the graph and the secant line connecting the intended endpoints. Students should not only be able to calculate the average rate of change, but they should also be able to generate a visual representation and use the visual representation to estimate the average rate of change over a specified interval. Students will gain a deeper conceptual understanding when they compare their estimations to the actual average rate of change for a non-linear function. As students solidify their understanding, they should be able to explain what the average rate of change means in the context of a problem when given symbolic representations, tables, graphs, or contextual situations. As students use multiple representations to evaluate the average rate of change, they should be able to explain the relationship between the multiple representations using both appropriate mathematical language and appropriate justifications.

### Level 4:

Students should extend their understanding of average rate of change by comparing the average rate of change of one interval to another interval of the same function. Students should also further their understanding by creating their own contextual situations and interpreting the average rate of change for a significant interval. Students should be intentional in determining which interval or intervals they select and explain the importance of the interval(s) with respect to the context using both precise mathematical vocabulary and precise justifications.

**Standard A1.F.IF.C.6 (Supporting Content)**

Graph functions expressed symbolically and show key features of the graph, by hand and using technology.

**A1.F.IF.C.6a** Graph linear and quadratic functions and show intercepts, maxima, and minima.

**A1.F.IF.C.6b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

**Scope and Clarifications:**

Tasks in A1.F.IF.C.6b are limited to piecewise, step and absolute value functions.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Represent a constant rate of change between two variables as slope of a line.</p> <p>Use characteristics of the symbolic representation of a function to determine the function type and behavior of the graph.</p> <p>Identify <math>y=x</math> as the parent function for linear function</p> <p>Explain the effects of slope &amp; intercepts on a linear function.</p>	<p>Graph a linear function by hand and using technology.</p> <p>Sketch the graph of a quadratic function given intercepts and extrema.</p> <p>Explain how the absolute value function is an example of a piecewise-defined function.</p> <p>Identify key features, such as extrema and intercepts, from a graph of a piecewise-defined function.</p> <p>Infer restrictions on the domain and range from a graph.</p> <p>Identify key features, such as intercepts and extrema, from a graph of a quadratic function.</p>	<p>Graph a linear function by hand and using technology and identify the slope and intercepts.</p> <p>Graph a quadratic function by hand and using technology identifying intercepts, maxima, and minima.</p> <p>Graph a piecewise-defined functions, including step functions and absolute value functions by hand and using technology.</p> <p>Attend to precision when illustrating intercepts, maxima, and minima and determine the domain and range of the function.</p>	<p>Explain the relationship that exists between a contextual problem and the key features of a graph for linear, quadratic, and piece-wise functions.</p> <p>Critique graphs drawn by others to ensure key features are shown efficiently and appropriately.</p> <p>Given a graph, write the corresponding function symbolically.</p> <p>Explain restrictions on domain and range in context.</p>

## Instructional Focus Statements

### Level 3:

In grade 8, students used functions to model relationships between quantities and construct a function to model a linear relationship. Students also determined if a function is linear or nonlinear, and they had experience interpreting and representing functions algebraically, numerically, graphically, and verbally. In Algebra I, students will be introduced to additional function families and more key features such as extrema and end behavior. Intercepts, shape, domain, and range take on greater meaning to students through the exploration of a variety of functions. Therefore, it is important for instruction to provide tasks that allow students to explore the behavior and varying parameters of functions. To meet the rigor of this standard, students should be given the opportunity to work with functions that vary in their symbolic representation. For example, students should experience point-slope, slope-intercept, and standard form of a linear function as well as standard form, factored form, and vertex form of a quadratic function. This will help students have access to the problem regardless of the symbolic representation which is further developed in standard A1.F.IF.C.7a in which students identify key features through algebraic manipulation.

Students should be able to graph functions by hand and with the use of technology. It is imperative to model how to graph with a graphing calculator or other graphing device. This is the first time students will use technology to graph a function type other than a linear function. Furthermore, ample time must be given for students to explore how a table of value can be helpful in identifying key features, domain, and range from a graph, which is supported by standard A1.F.IF.B.4. The use of technology allows students to explore functions whose key features are irrational values, which can be located with the use of a device.

This standard appears in Algebra II as well, students will be required to graph and interpret key features of additional function types. Therefore, instruction should introduce the concept of parent functions. This will help students make the connection of how transformations affect the graph, equation, and table of a function, which is explored in A1.F.BF.B.2. This standard can be integrated in instruction as students are presented with problem types whose symbolic representation varies and asked to identify the parent function and describe the transformation from its original, non-transformed graph. This will help students attend to precision as they graph functions of many types and use their understanding of transformations to support the reasonableness of their graph. Instruction should provide ample opportunity for students to compare and contrast the graphs of functions, and it should help them efficiently recognize a parent function when expressed symbolically and graph it fluently.

Students may struggle with domain restrictions and piecewise-defined functions. Technology can support student understanding of domain and range restrictions by graphing multiple functions on the same coordinate plane. For instance, have students graph  $y = x^2$ ,  $y = (x + 4)^2$ , and  $y = x^2 - 4$  simultaneously and make observations about the domain and range by looking at the graph and analyzing the table. Instruction should provide examples for students to discover the relationship between the algebraic representation and the resulting domain and/or range to help students understand piecewise-defined functions, instruction should begin by defining a simple piecewise function of two constant functions in context so they can make connections between the graph and the domain. Students will need to connect their prior knowledge of inequalities when working with domain in piecewise functions.

**Level 4:**

Students should extend their conceptual understanding of key features of graphs by connecting key features to the relationships that exist in contextual problems. Using their knowledge of multiple representations built in standard A1.F.IF.C.8, students should be able to provide a graph, table, equation, and verbal representations of a contextual situation. Instruction should include posing purposeful questions asking them to show and describe key features from their created problem in context. Students should be given the opportunity to look at graphs drawn by others so they can analyze and critique their peers work. Through the analysis of many graphs, students should develop an understanding of when key features are efficiently and effectively represented, and, if not, provide a suggestion for representing them more appropriately.

**Standard A1.F.IF.C.7 (Supporting Content)**

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

**A1.F.IF.C.7a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Explain the zeros in terms of the context, when given the zeros of a quadratic function embedded in a contextual problem.</p> <p>Explain the maximum or minimum in terms of the context, when given either the maximum or minimum of a quadratic function embedded in a contextual situation.</p>	<p>Explain the zeros in terms of the context, when given a factored quadratic equation embedded in a contextual problem.</p> <p>Explain the vertex in terms of the context, when given a quadratic function represented in vertex form.</p>	<p>Interpret the meaning of zeros, y-intercept, extreme value, and the axis of symmetry in the context of a real-world problem.</p> <p>Recognize which form of a quadratic function is most appropriate for revealing certain properties, when given a real-world problem.</p>	<p>Write a quadratic function from a real-world context given zeros or the extreme value, and one additional point.</p> <p>Create a real-world contextual problem that matches a given set of key features for a quadratic graph.</p>

**Instructional Focus Statements**

**Level 3:**

In previous grades, students converted linear functions to slope-intercept form to reveal two characteristics of linear functions: slope and y-intercept. In Algebra I, students look for and make use of structure to rewrite quadratic functions to reveal characteristics about their graphs. Therefore, this standard relies on a deep understanding of A1.A.SSE.B.3, in which students choose and produce equivalent forms of quadratic expressions. Teachers should build upon this by supporting students in converting among standard, vertex, and factored forms and understanding which form is most appropriate for revealing certain key features.

By completing the square, students can write quadratic functions in vertex form to reveal the vertex and axis of symmetry of the function. Visually, students see that the vertex is the maximum or minimum of the function, which is important in a variety of contexts. For example, it can reveal the unit

price that maximizes profit, the time when a projectile reaches its maximum height, or the length of a rectangle that maximizes the area.

Teachers should provide a variety of contexts in which the type of variables differ so that students can develop a deep understanding of how each component relates to the real-world context. Moreover, matching functions given in various forms with possible graphs that they could represent is a useful task to help students visualize how each form of a quadratic reveals key features of the function.

**Level 4:**

Students with a deep understanding of the three forms of a quadratic should be able to create quadratic functions when information about key features. For example, students could be given one zero and the vertex to write the quadratic function that it represents. In this example using the symmetry of the graph, teachers can focus students' attention on finding an additional zero to build factored form. From there, they can substitute in the  $x$  and  $y$  value of the vertex to find the leading coefficient,  $a$ . Teachers should create functions within a context so that students continue to see the key features of quadratic functions within various contexts. Additionally, students should further extend their understanding by creating real-world problems that match a given set of key features.

## INTERPRETING CATEGORICAL and QUANTITATIVE DATA (S.ID)

### A1.F.IF.C.8 (Supporting Content)

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

#### Scope and Clarifications:

- i) Tasks have a real-world context.
- ii) Tasks are limited to linear functions, quadratic functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify the y-intercept of a function from multiple representations.</p> <p>Identify the slope of a linear function from multiple representations.</p> <p>Describe connections among multiple representations of a linear function.</p> <p>Compare properties of two linear functions each represented in a different way.</p>	<p>Identify the zeros of a function from multiple representations.</p> <p>Identify the vertex of a quadratic function from multiple representations.</p> <p>Identify the percent rate of change of an exponential function from multiple representations.</p> <p>Describe connections among multiple representations of a quadratic function.</p> <p>Describe connections among multiple representations of an exponential function.</p> <p>Describe connections among</p>	<p>Compare properties of two exponential functions each represented in a different way.</p> <p>Compare properties of two piecewise-defined functions each represented in a different way.</p> <p>Compare properties of two quadratic functions each represented in a different way.</p> <p>Compare properties of two functions from different function families each represented in a different way.</p>	<p>Compare properties of two functions within a context.</p> <p>Use precise mathematical vocabulary to explain the relationships of the various representations of a function.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	multiple representations of a piecewise-defined function.  Move fluently among multiple representations of a function.		

### Instructional Focus Statements

#### Level 3:

Prior to comparing properties of two functions represented in different ways, students need to first identify properties of functions and make connections between different representations of the same function. Instruction should focus on students representing a function in multiple ways. As students translate between representations they must consider relationships among quantities and how each representation provides a unique perspective of the function. Teachers can foster this way of seeing mathematics by having students discuss the similarities among representations that reveal the key features of a function that persist regardless of the form. Through these discussions students can determine which representations are most appropriate for revealing certain key features of the function.

In grade 8, students compared properties of two linear functions each represented in a different way. Once students have a strong understanding of the various representations of linear, quadratic, piecewise-defined, and exponential functions in Algebra 1, they can begin to compare properties of two functions represented in different ways. For example, given a graph of one quadratic function and a table of another, a student should be able to compare their y-intercepts. One strategy that can sometimes be useful is to convert one or both to a different form so that both functions are represented the same way. As students begin to grasp this concept, it is important that teachers provide students with examples that include each function type, with some situated within a context. Therefore, comparing properties in different representations further supports students' understanding of each function type, which means this standard can be paired nicely with other standards that focus on properties and graphs of linear, quadratic, piecewise-defined, and exponential functions, such as A1.F.IF.B.3 and A1.F.IF.C.6. As students recognize various function types in multiple representations, discussion should lead to the comparison of functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a quadratic function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties.

**Level 4:**

Students with a deep understanding of the various function types and representations should also be able to compare functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a quadratic function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties. Once conclusions are formed, teachers can ask further questions related to the context. For example, given a graph of a linear function and an algebraic representation of a piecewise-defined function each describing the cost of a cell phone plan, decide which plan is better. Students should be given the opportunity to describe how to identify function types and compare the properties of functions in various forms. At this level, teachers should expect students to use precise mathematical vocabulary to describe and justify these relationships and qualities.

## Building Functions (F.BF)

### Standard A1.F.BF.A.1 (Supporting Content)

Write a function that describes a relationship between two quantities.

**A1.F.BF.A.1a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

### Scope and Clarifications: (Modeling Standard)

i) Tasks have a real-world context.

ii) Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Write a function defined by an expression to model a linear relationship, given a table or graph.</p> <p>Identify the independent and dependent variable in a real-world context.</p> <p>Identify the first term and rate of change of a linear function, given a table or graph.</p> <p>Identify the first term and common ratio of an exponential function, given a real-world context.</p>	<p>Determine whether a real-world context would be represented by a linear or non-linear function.</p> <p>Write a function defined by a recursive process or steps for calculation to model a linear relationship, given a real-world context.</p> <p>Write a function defined by a recursive process or steps for calculation to model an exponential relationship, given a real-world context.</p>	<p>Write a function defined by an expression to model a linear relationship, given a real-world context.</p> <p>Write a function defined by an expression to model a quadratic relationship, given a real-world context.</p> <p>Write a function defined by an expression to model an exponential relationship with domain in the integers, given a real-world context.</p> <p>Compare key characteristics of real-world contexts that can be described by various types of functions.</p>	<p>Create a real-world context that would generate the given function, given a function defined by an expression, a recursive process, or steps for calculation.</p> <p>Explain the various ways a function can be defined and in what real-world contexts they would be appropriate.</p> <p>Justify why a specific type of function should be used to describe a given real-world context.</p>

## Instructional Focus Statements

### Level 3:

In grade 8, students construct a function defined by an expression to model a linear relationship. In Algebra I, students create linear functions given a real-world context defined by a recursive process or steps for calculation in addition to an explicit expression. In many situations, it is natural to use a function defined recursively, which generates values by applying operations on previous terms. For example, mortgage payments and drug dosages can be described with a recursive process. Students also create exponential functions defined in the same three ways. Instruction should focus on creating functions given a real-world context, while recognizing appropriate ways to define functions.

If given a table of values, students should first recognize which type of function the table of values represents. Instruction should focus on the relationship between consecutive points to see if there is a common first difference or constant additive change (linear function), a common second difference (quadratic function), or a common ratio or constant multiplicative change (exponential function). Once students identify the function type, they should begin to write the function given the common first difference, second difference, or ratio and other information from the table.

To build coherence, it is important for students to make connections between linear functions and arithmetic sequences and between exponential functions and geometric sequences. Thus, instruction can be nicely paired with standard A1.F.LE.A.2, where students generate arithmetic and geometric explicit formulas to model situations. Both linear functions ( $y = ax + b$ ) and arithmetic sequences ( $a_n = a_1 + d(n - 1)$ ) describe additive changes, and students should make connections between the two. For example,  $b$  is equivalent to  $a_0$  and  $a$  is equivalent to  $d$ . Similarly, exponential functions ( $y = ab^x$ ) and geometric sequences ( $a_n = a_1 r^{n-1}$ ) both describe multiplicative changes and  $a$  is equivalent to  $a_0$  and  $b$  is equivalent to  $r$ . Students should understand the similarities, but instruction should also help students realize an important difference: arithmetic and geometric sequences are discrete while linear and exponential functions are continuous. This can be done by comparing the graphs of an arithmetic sequence and a linear function, for example.

### Level 4:

As students develop a deep understanding of this standard, they should be able to create a real-world scenario given a function or combination of functions. Moreover, they should be able to describe which characteristics of their scenario correspond to each part of the given function. For example, given  $y = 10 + 5X$  a student might create a scenario similar to the following: A ski lodge charges \$10 to rent a snowboard and \$5 for each hour it is used. In this function,  $x$  represents time in hours and  $y$  represents the final cost. The student should also be able to explain how the  $5X$  relates to \$5 per hour and how the 10 in the function relates to the initial charge of \$10. Similar scenarios can be created to describe exponential functions (e.g., sharing news or half-life).

### A1.F.BF.B.2 (Supporting Content)

Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

#### Scope and Clarifications:

- i) Identifying the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative) is limited to linear, quadratic, and absolute value functions.
- ii)  $f(kx)$  will not be included in Algebra 1. It is addressed in Algebra 2.
- iii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, absolute value, and exponential functions with domains in the integers.
- iv) Tasks do not involve recognizing even and odd functions.

#### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Describe, using precise mathematical vocabulary, transformations that would map a geometric figure to its image.</p> <p>Write the function defined by <math>f(x) + k</math>, given the function and a positive value of <math>k</math>.</p> <p>Write the function defined by <math>kf(x)</math>, given the function and a positive value of <math>k</math>.</p> <p>Write the function defined by <math>f(x + k)</math>, given the function and a positive value of <math>k</math>.</p>	<p>Compare <math>f(x)</math> and <math>f(x) + k</math> and illustrate an explanation of the effects on the graph using technology.</p> <p>Compare <math>f(x)</math> and <math>kf(x)</math> and illustrate an explanation of the effects on the graph using technology.</p> <p>Compare <math>f(x)</math> and <math>f(x + k)</math> and illustrate an explanation of the effects on the graph using technology.</p>	<p>Describe the effect on the graph for specific values of <math>k</math>, given two functions, <math>f(x)</math> and <math>f(x) + k</math>.</p> <p>Describe the effect on the graph for specific values of <math>k</math>, given two functions, <math>f(x)</math> and <math>kf(x)</math>.</p> <p>Describe the effect on the graph for specific values of <math>k</math>, given two functions, <math>f(x)</math> and <math>f(x + k)</math>.</p> <p>Determine the value of <math>k</math> for a specific vertical or horizontal translation or vertical stretch or compression, given two graphs, the image and pre-image.</p>	<p>Write the equation of a function given the graph by identifying the transformation(s) to the parent function.</p> <p>Apply transformations to a function that has already been transformed.</p> <p>Explain why changes to the argument of <math>f(x)</math> affect the input values and changes outside the function affect the output values.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Describe multiple effects on a graph for specific values of $a$ , $h$ , and $k$ given two functions, $f(x)$ and $af(x + h) + k$ .	

### Instructional Focus Statements

#### Level 3:

In grade 8, students verify experimentally the properties of rotations, reflections, and translations of simple figures. Students expand on this concept in Algebra I by applying transformations to functions and describing the transformations using function notation. Students must have a deep understanding of function notation and how to write a function defined by function notation (e.g., given  $f(x)$ , write the function  $f(x + 3)$ ). Therefore, pairing A1.F.BF.B.2 with A1.F.IF.A.2 will allow students to see how function notation can be used to generalize transformations to all functions.

To understand how  $a$ ,  $h$ , and  $k$  impact the graph of  $f(x)$  when compared to  $af(x + h) + k$ , students can use technology (i.e., calculator or online graphing tool) to experiment with  $f(x) + k$ ,  $af(x)$ , and  $f(x + h)$ . As students vary one value at a time, they can begin to discern how each component affects the graph of  $f(x)$ . Careful attention should be made to why  $h$  translates the graph  $-h$  units horizontally. One explanation is to compare  $f(x)$  with  $f(x - 5)$  and see that  $f(3)$ , for example, will produce the same output value as  $f(8 - 5)$ . In this example, an input of 3 into  $f(x)$  is equivalent to an input of 8 into  $f(x - 5)$ , and 8 is 5 units to the right of 3, not left. Students can think about it as having to undo what has been done to  $x$  inside the argument, which leads nicely to understanding  $f(kx)$  in future courses. Meanwhile,  $f(x)$  represents the output values. So, any operations performed to  $f(x)$  outside the argument only affect the  $y$  values, which results in vertical transformations. Once students understand the effects of each component individually, they should then attempt to describe changes to a graph involving multiple transformations at once.

Additionally, connections between A1.F.BF.B.2 and A1.A.SSE.B.3 should be made. Converting to the vertex form of a quadratic reveals the transformations being made to  $y = x^2$ , which allows students to easily locate the vertex and determine the concavity of the parabola. Making this connection will help support a deep conceptual understanding of vertex form. Students should realize that the same transformations will be applied to other function types such as polynomials and trigonometric functions in future courses.

#### **Level 4:**

Students with a deep understanding of this standard should be able to write the equation of a function given the graph by identifying the transformation(s) to the parent function. Teachers should focus students' attention on the order of each transformation. For example, given  $-f(x) + 9$ , the graph is first reflected across the x-axis, then shifted up 9, rather than shifted up 9 then reflected across the x-axis due to the order of operations. It is also important that teachers place an emphasis on factoring out b from inside the argument so that the horizontal shift can be found. For example, write  $(2x - 6)^2$  as  $(2(x - 3))^2$  instead, revealing a horizontal shift of 3 to the right, not 6. Teachers should help students understand that transformations can be made to functions that have already been transformed. For example, given  $f(x) = (x + 2)^2$ , write an equation for  $f(x - 5)$  and describe the overall change in the graph. In addition, students at this level should be able to explain concepts such as why changes inside the argument of a function have the inverse effect on a graph. Taken collectively, these students should understand a function as a process that generates output values from particular input values. Building on this understanding, students should make connections with which transformations perform operations to the input values prior to the function's operations and which transformations perform operations to the output values after the function has been applied.

## Linear, Quadratic, and Exponential Models (F.LE)

### Standard A1.F.LE.A.1 (Supporting Content)

Distinguish between situations that can be modeled with linear functions and with exponential functions.

**A1.F.LE.A.1a** Recognize that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

**A1.F.LE.A.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

**A1.F.LE.A.1c** Recognize situations in which a quantity grows or decays by a constant factor per unit interval relative to another.

### Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Recognize a function as linear, both from a graph and an equation.</p> <p>Recognize a function as exponential, both from a graph and an equation.</p>	<p>Find slope of a line from a graph, table of values, or given two coordinate points.</p> <p>Recognize that a line has a constant rate of change.</p> <p>Recognize that an exponential function does not have a constant rate of change</p>	<p>Recognize that linear functions have a constant rate of change, while exponential functions do not.</p> <p>Informally show or explain that linear functions grow by adding the same number per unit. This should be done algebraically, graphically, and using words in context of a real-world application.</p> <p>Informally show or explain that exponential functions grow by multiplying the same factor per unit. This should be done algebraically, graphically, and using words in context of a real-world application.</p>	<p>Prove using precise mathematical language that a linear function grows by adding the same number per unit, while an exponential function grows by multiplying the same factor per unit.</p> <p>Create a real-world example of a situation that can be modeled by a linear function and explain why it is linear.</p> <p>Create a real-world example of a situation that can be modeled by an exponential function and explain why it is exponential, including why it is growth or decay.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		<p>Determine if a given real-world situation has a constant rate of change and can be modeled by a linear function.</p> <p>Determine if a given real-world situation can be modeled by an exponential function.</p> <p>Determine if a given real-world situation that can be modeled by an exponential function represents growth or decay.</p>	

### **Instructional Focus Statements**

#### **Level 3:**

Students first learn how to find slope of a line in grade 7. In grade 8, they begin exploring non-linear functions. In Algebra I, students compare the behavior of linear and exponential functions in terms of rate of change and its effect on end behavior.

As this is a modeling standard, students should be working with linear and exponential situations embedded in real-world applications. Allow students to explore the rate of change of linear and exponential functions from graphs, tables of values, equations, and real-world examples to determine if the rate of change remains constant. Slope should be calculated over equal intervals and compared. This may be easier if students organize the information in a table.

It is important for students to be able to identify when a function is linear vs. exponential, therefore, students should be provided with mixed examples and not just one or the other in isolation.

Students should be expected to use mathematical structure and repeated reasoning through multiple representations to see that the rate of change for a linear function remains constant.

Students may struggle with finding the percent growth or decay in an exponential function. For example, the amount of money earned in an investment is represented by the function  $f(x) = 1500(1.07)^t$ . By completing a table of values, students can see that the function represents growth, but they may be confused by the 1. It may be easier for them to understand where the 1 comes from in an example of buying a pair of blue jeans at a store. Show students that they can calculate the cost of the blue jeans and add sales tax in one step by taking 100% of the cost of the blue jeans plus the percent of sales tax. For a \$20 pair of blue jeans at 8% sales tax, they can multiply 20 by 1.08 (100% + 8%) to get the total out of pocket cost. Likewise, if the blue jeans are on sale at 15% off, they can multiply 20 by .85 (100% - 15%) to get the sale price.

#### **Level 4:**

The focus of this standard is on the comparison of the rates of change between linear and exponential functions. Students should be challenged to prove that functions that grow by adding the same number are linear as compared to functions that grow by multiplying the same factor are exponential. Students should attend to precision and use appropriate mathematical language in their argument.

To ensure students are making connections between real-world situations and linear functions, have them create their own real-world examples that can be represented by linear functions. They should also explain why the situation is linear.

To ensure students are making connections between real-world situations and exponential functions, have them create their own real-world examples that can be represented by exponential functions. Students should represent their examples in words, algebraically, in a table, and graphically. They should include in their explanation why the situation represents exponential growth or decay.

**Standard A1.F.LE.A.2 (Supporting Content)**

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.

**Scope and Clarifications: (Modeling Standard)**

Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Recognize a graph of a function as linear.</p> <p>Recognize a graph of a function as exponential.</p>	<p>Recognize a function as linear from a table, a description, or a set of ordered pairs and justify it using rates of change.</p> <p>Recognize a function as exponential from a table, a description, or a set of ordered pairs and justify it using rates of change.</p>	<p>Construct a linear function given a graph.</p> <p>Construct a linear function given a table of values.</p> <p>Construct a linear function given a description of a simple real-world relationship.</p> <p>Construct a linear function given a set of input-output pairs (ordered pairs).</p> <p>Construct an exponential function given a graph.</p> <p>Construct an exponential function given a table of values.</p> <p>Construct an exponential function given a description of a simple real-world relationship.</p>	<p>Analyze functions created by others to determine accuracy and explain and correct any errors.</p> <p>Create a real-world situation that may be modeled by a linear function and write the function.</p> <p>Create a real-world situation that may be modeled by an exponential function and write the function.</p> <p>Collect data for a real-world situation that can be represented by a linear or exponential function and write the function that models it. Define the variables and explain in context why the function models the situation.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Construct an exponential function given a set of input-output pairs (ordered pairs).  Construct a function given an arithmetic or geometric sequence or a description of one.	

### Instructional Focus Statements

#### Level 3:

This standard aligns with several Algebra I standards, including the CED, IF, BF, and SSE clusters, and should be integrated with those rather than as a stand-alone lesson. Once students have been exposed to both linear and exponential functions, instruction should include examples of both function types in multiple representations including graphs, tables, and descriptions to allow students time to determine whether the function is linear or exponential as well as write the function. This is a good opportunity to relate input and output values with independent and dependent variables.

As this is a modeling standard, real-world examples should be provided and students should be required to explain how the function models the context. This standard also allows for students to connect their learning about arithmetic and geometric sequences to creating the functions that model them. These should also come from multiple representations including graphs, tables, and descriptions of real-world situations. In Algebra I, the examples should be limited to simple single-step context. In Algebra II, the examples will increase in complexity and incorporate multi-step context.

#### Level 4:

To increase the level of understanding, students should critique examples of functions created by others. One way to do this is to have students work in pairs with one student constructing a function and the other student checking it. For example, student A constructs a function from a graph while student B constructs a function from a table of values. Then they swap papers and student B graphs student A's function while student A creates a table of values from student B's function. Then each compares their work with the originals. If they do not match, they must determine where the mistake was made and correct it.

Once students have a good understanding of constructing both linear and exponential functions, they can create their own real-world examples. This can also involve students predicting a situation that would provide data that could be modeled by a linear or exponential function, collecting that data, and

writing the function based on that data to test their prediction. Students should show and explain why the real-world example represents a linear or an exponential function, including representing it in multiple ways.

**Standard A1.F.LE.A.3 (Major Work of the Grade)**

Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

**Scope and Clarifications: (Modeling Standard)**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Understand what is meant by an increasing function. Describe numerically why an increasing graph visually rises.</p> <p>Identify the interval(s) where a function is increasing and decreasing given a graph or a table of values.</p>	<p>Calculate the average rate of change of linear, quadratic, polynomial, and exponential functions over a given interval.</p> <p>Describe interval(s) where a function is increasing and decreasing using interval notation or inequality notation given a graph or table of values.</p>	<p>Compare the end behavior of graphs of lines, quadratics, polynomials, and exponentials to determine which increases faster.</p> <p>Find and compare the average rate of change of lines, quadratics, polynomials, and exponentials over equal intervals and make conclusions.</p> <p>Defend why a quantity increasing exponentially will eventually exceed a linear, quadratic, or polynomial function and justify their conclusion by testing values.</p>	<p>Verify and explain why a quantity in one function type will eventually exceed a quantity in another function type.</p> <p>Observe graphs that model real-world scenarios and explain in context the reasonableness of why one graph increases faster than the other.</p> <p>Find the exact quantity where an exponential function exceeds another using technology and explain what it means in context of the real-world situation.</p>

**Instructional Focus Statements**

**Level 3:**

Students should be provided examples of different types of functions in multiple representations to compare which type function will increase faster. It is important that students understand what is meant by "increasing faster". This will typically be easier for students to see in a table of values with the independent variable values listed in numerical order.

When analyzing graphs, students should be instructed to read the graph left to right and can consider end behavior to help them draw a conclusion. As this is a modeling standard, it is important that students interpret the graphs in context to understand the relationship between the variables.

Students have more experience with linear functions and should be able to tell if it is increasing or decreasing since the rate of change remains constant. To help students understand the rate at which a function is increasing for functions other than linear, they should compare rates of change over equal intervals for the functions being compared. Therefore, standard A1.F.IF.B.5 is a pre-requisite standard for this standard. Organizing this data in a table will help students draw conclusions.

Comparing graphs of functions on the same coordinate plane will also help students see what "increasing faster" looks like. If students struggle with getting the two graphs confused when they are on the same coordinate plane, have them graph them with different colors, or put one on tracing paper so that it can be placed over the other one. This can also be alleviated by having students compare the graphs using technology.

#### **Level 4:**

At this level of understanding, students should be able to explain precisely how they know a quantity that increases exponentially will eventually exceed that of a quantity that increases linearly, quadratically, or in another polynomial function. Students should also be able to explain in context the reasonableness of why a graph that models one situation would increase faster than a graph that models another situation. Allowing students to work in pairs or small groups when analyzing pairs of functions will provide the opportunity to experience new approaches to their thinking and a deeper understanding of the concepts.

**A1.F.LE.B.4 (Supporting Content)**

Interpret the parameters in a linear or exponential function in terms of a context.

**Scope and Clarifications: (Modeling Standard)**

For example, the total cost of an electrician who charges 35 dollars for a house call and 50 dollars per hour would be expressed as the function  $y = 50x + 35$ . If the rate were raised to 65 dollars per hour, describe how the function would change.

- i) Tasks have a real-world context.
- ii) Exponential functions are limited to those with domains in the integers.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define slope as a rate of change.</p> <p>Identify the slope and the y-intercept in a linear function written in slope-intercept form.</p> <p>Identify the coefficient, base, and exponent in an exponential function.</p>	<p>Determine the slope and y-intercept of a linear function in a graph.</p> <p>Calculate the slope of a line that passes through two given points.</p> <p>Calculate the y-intercept of a linear function algebraically.</p> <p>Identify the initial value of an exponential function in a graph.</p> <p>Calculate the initial value of an exponential function algebraically.</p> <p>Calculate the growth rate of an exponential function by finding the ratio of successive terms.</p>	<p>Explain the meaning of the slope and y-intercept in context of the real-world situation, given a linear function.</p> <p>Given an exponential function with a domain in the integers, explain the meaning of the coefficient, the base, and the exponent in context of the real-world situation.</p> <p>Predict and determine how a linear function is affected by a change in the slope or y-intercept. Explain this change in context.</p> <p>Predict and determine how an exponential function is affected by a change in the coefficient, base, or exponent. Explain this change in context.</p>	<p>Reflect and respond to the explanations given by others.</p> <p>Create a real-world scenario that can be modeled by it, given a linear function.</p> <p>Create a real-world scenario that can be modeled by it, given an exponential function.</p>

## **Instructional Focus Statements**

### **Level 3:**

This standard is an extension of A1.A.SSE.A.1, in which students interpret parts of expressions. Instruction should focus on how the different components affect each other. As this standard is a modeling standard, examples should connect to a real-world context. Use questions that ask students to interpret the slope and y-intercept of linear functions in the context of a real-world situation. Likewise, ask students to interpret the coefficient, base, and exponent of exponential functions in context of a real-world situation. Then extend their learning by asking them to determine the effect of changes to the parameters on the function. The scope provides an example of a linear function question. An exponential example might be: given an account that is modeled by  $A(t) = 200(1.005)^{12t}$ , determine the initial amount invested, the interest rate, and how often the money is compounded. Then describe what the effect would be if the initial investment is increased by \$100.

Students need ample opportunities working with contextual problems. For example, students should be able to make a prediction of the effect of a change in a parameter and then verify if their prediction was correct by applying the change and comparing the results. Repetition of this activity will help students develop a better understanding of the properties of the operations within the function. Connecting the function to the context will help them justify their reasoning for their predictions. This repetition will also help students see the structure of the function and make a connection to its use as a general formula for the given real-world situation.

It is important for students to attend to precision in their interpretations and explanations should include units to ensure they are interpreting completely and correctly.

### **Level 4:**

Students should have opportunities working with examples of linear and exponential functions and ask them to create a real world scenario that could be modeled by them. For example, students should be able to critique others' interpretations of the parameters and correct any mistakes. Students should look for correct and precise mathematical language in others' explanations, explain any mistakes made or lack of precision, and provide accurate corrections. This standard can be easily integrated with A1.F.IF.B.3 and interpreting key features of a graph in connection with the function and the context.

## Interpreting Categorical and Quantitative Data (S.ID)

### Standard A1.S.ID.A.1 (Supporting Content)

Represent single or multiple data sets with dot plots, histograms, stem plots (stem and leaf), and box plots.

#### Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course

### Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Calculate the median of a data set.</p> <p>Organize given data values onto an already created stem and leaf plot.</p> <p>Represent data on a histogram with given scale and axes.</p> <p>Apply correct labels for components and/or axes when representing data graphically.</p>	<p>Represent a single data set with a dot plot.</p> <p>Represent a single data set with a histogram.</p> <p>Represent a single data set with a box plot.</p> <p>Represent a single data set with a stem and leaf plot.</p> <p>Choose an appropriate scale for the graph when representing data graphically.</p>	<p>Create parallel or side-by-side box plots or histograms with the same scale.</p> <p>Determine which type of data plot would be most appropriate for a set of data.</p> <p>Use real-world data (represented in a table) to create dot plots, histograms, stem plots, or box plots.</p> <p>Use technology to represent single or multiple data sets with dot plots, histograms, stem plots (stem and leaf), and box plots.</p>	<p>Explain advantages and disadvantages of displaying data using different representations.</p> <p>Recognize misrepresentation of data in various types of plots.</p> <p>Explain what changes could be made to improve the representation of the data.</p>

## **Instructional Focus Statements**

### **Level 3:**

In grade 7, students interpret dot plots and box plots and use them to informally compare two data sets. In Algebra I, students use real-world data to create dot plots, histograms, or box plots, apply correct labels for components and/or axes, and choose an appropriate scale for the graph. Teachers should emphasize attending to precision as students label their axes, choose an appropriate scale, and specify the units of measurement so that others can easily understand their plots and histograms. Given two data sets, students should create parallel box plots or histograms with the same scale to, along with A1.S.ID.A.2, compare the center and spread of two or more data sets. Teachers should provide examples of parallel box plots or histograms that do not have the same scale to reveal how these comparisons can be misleading.

As students become comfortable creating each type of data plot, teachers should help students determine which type of plot is most appropriate for a given set of data. Teachers should have students consider how large the data set is, the format of the given data (values or frequencies), and the purpose of plotting the data. In A1.S.ID.A.2 and later in Algebra II, students will use their knowledge of histograms to understand the normal distribution curve, a continuous probability distribution.

### **Level 4:**

As students develop a deep understanding of this standard, they should realize the advantages and disadvantages to each type of representation and be able to explain them using precise mathematical vocabulary. For example, stem and leaf plots make it easy to find the median of a data set. However, stem and leaf plots are not very informative for small data sets. Box plots are great for identifying outliers and provide a nice summary of the data, but the exact data values are not retained.

As mentioned in level 3, teachers should provide examples of comparisons that are misleading due to different scales. To deepen this understanding, instruction should focus students' attention on recognizing misrepresented data and explaining how the visual representation of the data can be altered. Various components should be considered such as, scale, labels, units of measurement, breaks in the axes, or when a less appropriate plot or histogram is used to represent the data.

**Standard A1.S.ID.A.2 (Supporting Content)**

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Determine the mean, median, and interquartile range of a single set of data from a table.</p> <p>Define center and spread, when related to a data set.</p>	<p>Interpret and compare the mean, median, and interquartile range in the context of a data set.</p> <p>Calculate the mean, median, interquartile range, and standard deviation of a data set using technology.</p>	<p>Explain similarities and differences using specific measures of central tendency and measures of dispersion, given two or more data sets.</p> <p>Determine within how many standard deviations above or below the mean a data value is.</p>	<p>Compare and contrast multiple data sets using measures of center and spread.</p> <p>Explain how standard deviations can be used to describe how much of the data is around the mean.</p>

**Instructional Focus Statements**

**Level 3:**

One of the major goals of statistics is to summarize, compare, and predict. In Grade 7, students informally compare the center and spread of two data sets represented in a dot plot or box plot. In Algebra I, students formalize their understanding by using interquartile range and standard deviation to compare the spreads of multiple data sets represented visually or in a table. Students should be able to discuss which data set has a greater average or typical value and which data set has greater variability, when using appropriate measures of center and spread. When data is approximately normal or is intended to represent the population mean, students should use mean and standard deviation. However, the median and interquartile range better represent data that is strongly skewed. Using statistics appropriate to the shape of the data will allow students to represent their data sets accurately and make stronger comparisons between multiple data sets within given contexts.

To aid students in making valid comparisons, teachers should provide multiple data sets with equal centers but different measures of spread and vice versa. In addition, teachers should provide data sets in which appropriate measures of center and dispersion vary so that students have opportunities to

justify parameters based on the shape of the given distributions. Thus, instruction can be nicely paired with A1.S.ID.A.3, in which students explain the advantages and disadvantages to using each parameter and how outliers impact the mean and median. In this course, students informally making inferences between data sets, while in future courses, they will use statistical tests (i.e., t-test) to determine if there is a significant difference between the means of two data sets.

**Level 4:**

Students should be able to compare and contrast data sets based on real-world situations. Therefore, students should think about how the differences in center and spread relate to the real-world context and how the information can inform decision making. Once the conclusions are made, teachers should help students write their results using precise mathematical vocabulary.

In addition, students at this level should use the empirical rule of approximately normal distributions to tell what percent of data values fall within whole-numbered standard deviations from the mean. For example, given wait times at a particular restaurant have a mean of 15 minutes with a standard deviation of 5 minutes, what percent of the wait times are between 10 and 20 minutes? In this case, 68% of the wait times would be between 10 and 20 minutes. In Algebra II, students will continue to use the empirical rule to investigate more complex problems such as: what percent of wait times are below 30, above 5, or between 10 and 25?

**A1.S.ID.A.3 (Supporting Content)**

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify unusual data points in a data set.  Know that standard deviation uses the mean and that interquartile range uses the median.	Recognize and name different distribution shapes and describe their center, shape, and spread.  Identify outliers in a data set.	Choose which measure(s) are most appropriate for comparison based on the shape of the distribution.  Describe the impact of an outlier on the center and spread of a data set.	Justify which measure(s) are most appropriate for comparison based on the shape of the distribution.  Explain advantages and disadvantages of using each measure of center and spread.  Explain how an outlier affects the mean and median differently.

**Instructional Focus Statements**

**Level 3:**

Students should be expected to recognize the advantages and disadvantages of using each measure of center and spread and how outliers impact each. A common misconception can occur regarding the labeling of outliers. Some students identify data points that are not typical and assume they are outliers. However, identifying these points as outliers prior to performing the appropriate calculations is misguided. Teachers should address this misconception carefully and consider using the term unusual or extreme data points in these situations prior to conducting the calculations.

As students are asked to discuss data values, teachers should focus on the relationship of the outlier to the skew of the data. Teachers should lead students to discover that the mean and standard deviation are less useful with strongly skewed data sets and the importance of using the median and interquartile range because they are not impacted by extreme data values. As students use the interquartile range to quickly identify outliers (data values

more than one-and-a-half times the IQR distance below the first quartile or above the third quartile), they should then be asked to compare the median to the quartiles and come to conclusions about the set of data based on these relationships.

This standard can be taught together with A1.S.ID.A.2, in which students compare multiple data sets, as the parameters used in the comparisons are largely determined by the impact outliers has on each measure of center. The focus of A1.S.ID.A.3 is to develop a deep conceptual understanding of why median and interquartile range are more appropriate for skewed data sets and why the mean and standard deviation are best used with approximately bell-shaped distributions, which occur naturally in many situations (e.g., height, weight, or strength of adults).

**Level 4:**

Students at this level should be able to explain how the mean and median are affected by a skew or an extreme value. Teachers can show students a distribution with its mean labeled with and without an extreme value to see how the mean is impacted by including this value. Teachers should ask students to explain how the mean shifts towards a skew or extreme value and explain why this tendency is the reason mean is not an appropriate in these situations.

To better understand this idea, teachers can have students compare the median and mean visually. The median of a density curve is the value that splits the area of the distribution in half, while the mean is the balance point (the point at which a fulcrum can be placed). A skew or extreme value affects the balance point (mean) much more than the halfway point of the area.

**Standard A1.S.ID.B.4 (Supporting Content)**

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

**A1.S.ID.B.4a** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.

**A1.S.ID.B.4b** Fit a linear function for a scatter plot that suggests a linear association

**Scope and Clarifications:**

Emphasize linear models, quadratic models, and exponential models with domains in the integers.

For A1.S.ID.B.4a:

- i) Tasks have a real-world context.
- ii) Exponential functions are limited to those with domains in the integers.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Choose a linear function to fit a given data set.</p> <p>Choose if a given scatter plot is best represented by a linear or exponential function.</p>	<p>Choose an exponential function that fits a given data set, where exponential functions are limited with domains in the integers.</p> <p>Use a given linear function to solve a problems in the context of the data.</p> <p>Fit a linear function to a given set of data.</p> <p>Create a line of best fit and discuss reasons for choosing the line, given a scatter plot.</p>	<p>Fit an exponential function to a given set of data, where exponential functions are limited to domains in the integers.</p> <p>Solve problems using a linear or exponential function in the context of the data, where exponential function are limited to domains in the integers.</p> <p>Describe the similarities and differences between their chosen line of best fit and the line of best fit created using technology, given a scatter plot.</p>	<p>Create a contextual situation with an embedded data set derived from a given function. Explain the relationship between the function, data set, and the contextual situation using precise mathematical language and justifications.</p> <p>Use a given function to explain the relationship between two quantities in a created context.</p> <p>Explain the difference between association and causation, given a set of data within context that suggests a linear relationship.</p>

**Instructional Focus Statements**

### **Level 3:**

In Grade 8, students developed an understanding of how to create a scatterplot, evaluate the scatterplot in order to describe any pattern associations between the two quantities, and informally fit a straight line to data when it visually resembled a straight line. In high school, students should extend this understanding to summarize, represent, and interpret data on two categorical and quantitative variables. This allows students to use mathematical models to capture key elements of the relationship between the two variables and explain what the model tells about the relationship. Students should gain a conceptual understanding of how to draw conclusions in addition to finding the equation for the line of best fit. As students' progress through Algebra it should become apparent to them that many real-world situations produce data that can be modeled using functions that are not linear. The exposure to quadratic and exponential functions broadens the options students have for modeling data sets, where data sets can be represented in tabular, graphical, or as a discrete set of points.

Students should be exposed to real-world situations where it is apparent that the scatter plot suggests a pattern that is more curved than linear in its visual depiction. Thus leading the student to realize that a linear function does not provide the closest fit to the data causing the student to consider other function types. It is imperative that students discover that sometimes obvious patterns may not tell the whole story. Students should develop an understanding that sometimes curves fit better than lines. Students should not only discover this algebraically but also develop an understanding of the connection that exists between the model and the contextual situation that it represents and understand that this connection is essential in identifying and building appropriate models. As students solidify their understanding, they should be able to describe how the variables are related within the context of the situation. Students should also use various forms of technology to explore and represent scatterplots as this will enhance their ability to see the relationship that exists between the variables.

### **Level 4:**

As students extend their understanding, they should be able to create a contextual situation with an embedded data set derived from a given function. Students should also be able to explain and provide justifications for the relationships that exist between the function, data set, and the contextual situation using precise mathematical language. Particular attention should be put on creating situations that differentiate between linear, quadratic, and exponential functions. Students should be able to explain why one function is more appropriate than another function for the contextual situation.

**A1.S.ID.C.5 (Major Work of the Grade)**

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify the slope and y-intercept, given a linear function in slope-intercept form.</p> <p>Identify two points on a scatterplot that could be used to build the line of best.</p>	<p>Estimate the slope and y-intercept of a linear model that would best fit data on a given scatterplot.</p> <p>Identify the slope of a linear model.</p> <p>Identify the y-intercept of a linear model.</p>	<p>Interpret the slope of a linear model in the context of the data.</p> <p>Interpret the y-intercept of a linear model in the context of the data.</p>	<p>Justify the appropriateness of the slope and y-intercept of a linear model in the context of the data.</p> <p>Explain why a linear model may only represent data in context within a certain domain.</p> <p>Explain why extrapolation may be unreliable for making predictions.</p>

**Instructional Focus Statements**

**Level 3:**

In grade 8, students developed an understanding of how to build a scatterplot, describe any patterns, and informally fit a straight line to data when it resembled a straight line. In high school, students should be expected to summarize, represent, and interpret data on two categorical and quantitative variables. Instruction should expose students to mathematical models and ask them to capture key elements of the relationship between the two variables and explain what the model tells about the relationship. Specifically, students should be interpreting key features of linear models such as slope and y-intercept. Instruction should focus on slope as the rate of change of the function, specifically identifying for every increase of one by the independent variable (i.e.,  $x$ ), the dependent variable (i.e.,  $y$ ) increases or decreases by the given slope. Discussion and questioning should be focused on interpretation of the y-intercept as the value of the dependent variable when the independent variable is zero and that when the independent variable is a unit of time, the y-intercept can be interpreted as the initial value of the dependent variable. For example, given a linear model of  $f(t) = 13.04t + 6.79$  where  $t$  represents years since 2009 and  $f(t)$  represents height of a tree in inches students should interpret 13.04 as the number of inches the tree grows per year and 6.79 inches as the initial height of the tree in 2009. Students should be expected to interpret values such as  $f(12)$ ,  $f(0)$ , or solve equations

like  $f(t) = 0$ . Overall, students should practice modeling with mathematics by using linear models to describe how one real-world quantity of interest depends on another, which will then be expanded to include other function types in future courses.

**Level 4:**

As students develop a deep understanding of linear models, they should be expected to justify the reasonableness of their model to determine if it makes sense in the given context. Given the example in level 3, students would consider if  $13.04x + 6.79$  is reasonable for describing tree growth, but students should be challenged to relate life experiences and number sense or research on the internet to make and explain this determination. Teachers should provide examples of scatterplots that do not fit the given context so that students have opportunities to justify why the model does not appropriately fit the context. For example, if the tree model was  $y = 275.23x - 5.47$  instead, students should realize that the slope is much larger than expected and that the tree had a negative height in 2009. Although some students might say that the tree was in the ground at that point, the height of the tree would still be positive.

As students apply linear models to make predictions about unknown situations, they should justify what domain is appropriate based on the given context. For example, if time is the independent variable, students should recognize that the domain should be restricted to values greater than or equal to zero. Teachers should also expect students to justify why extrapolation is not reliable in some situations and that it cannot be assumed that the existing trend will continue to unknown values beyond the given data.

**Standard A1.S.ID.C.6 (Major Work of the Grade)**

Use technology to compute and interpret the correlation coefficient of a linear fit.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Describe the correlation of the data set that it represents as strong or weak, given a correlation coefficient,</p> <p>Describe the correlation of the data set as having a positive or negative direction, given a graph.</p>	<p>Choose a correlation coefficient that represents a data set given a graph suggesting a linear fit.</p> <p>Choose the strength and direction that describes the relationship, given graph that represents a data set.</p>	<p>Using technology, calculate the correlation coefficient of a linear fit in mathematical problems.</p> <p>Interpret the correlation coefficient of a linear fit in mathematical problems.</p>	<p>Using technology, calculate and interpret the correlation coefficient of a linear fit in real-world problems.</p> <p>Create a real-world linear situation and calculate and interpret the correlation coefficient. Explain what the correlation coefficient represents with respect to the contextual problem, using mathematical precise vocabulary and justifications.</p> <p>Determine a situation in which it is predicted that there is a strong linear relationship between two varying amounts. Create an experiment in order to collect relevant data. Test the hypothesis by generating a line of best fit and calculating and interpreting the correlation coefficient.</p>

## **Instructional Focus Statements**

### **Level 3:**

Students developed a conceptual understanding of what it means for a set of data to have a linear association in grade 8. In Algebra I, students should extend their understanding to determine how strong the relationship is between the variables. Additionally, they should develop an understanding of the relationship between the direction of the fitted line and the data collected. Students will have had experience determining a linear model for a set of data. Here, they develop an understanding of the correlation coefficient as the relative closeness of the points as a set to the line of best fit. Students should understand that the correlation coefficient, denoted by  $r$ , measures the “tightness” of the data points about a line fitted to data, with a range of  $-1 \leq r \leq 1$ . Students should interpret this as the closer the  $|r|$  is to 1 the stronger the correlation of the points and the closer the  $|r|$  is to 0 the weaker the correlation of the points on the line. Students should also understand what this means with respect to the graphical nature of the scatter plot and the line of best fit. Students should be able to calculate the correlation coefficient, using technology, and understand that this value indicates that the data set has a correlation that can be described as strong, weak, positive or negative. This standard specifically states to calculate the correlation using technology rather than using a formula. The focus should be on interpreting the correlation coefficient with respect to the data set and contextual situation. Students should develop a conceptual understanding by analyzing graphical representations that illustrate what the correlation coefficient represents with respect to the graph as the distance each point is from the line of best fit. Students should also understand what this means with respect to the graphical nature on the scatter plot and the line of best fit. Additionally, students should explain in written and verbal form the connection between the correlation coefficient and contextual situation that it represents.

### **Level 4:**

As students extend their understanding of what a correlation coefficient means and tells about the data set, they should be able to illustrate and explain the relationship between the points in the scatter plot and the line of best fit. Students should also be able to create their own contextual situations and compute the correlation coefficient showing and comparing various correlations and explain the reasoning for the nature of the correlation based on the context. The emphasis on technology is specifically called out in the standard and students should use technology to not only compute the correlation coefficient but also use technology as a means to compare different models and their respective features. Additionally, students should solidify their understanding by reasoning and making sense of different correlation coefficients and their relationship to their contextual situations.

**A1.S.ID.C.7 (Major Work of the Grade)**

Distinguish between correlation and causation.

**Scope and Clarifications:**

There are no assessment limits for this standard. The entire standard is assessed in this course.

**Evidence of Learning Statements**

<b>Students with a level 1 understanding of this standard will most likely be able to:</b>	<b>Students with a level 2 understanding of this standard will most likely be able to:</b>	<b>Students with a level 3 understanding of this standard will most likely be able to:</b>	<b>Students with a level 4 understanding of this standard will most likely be able to:</b>
<p>Given a correlation coefficient, describe the correlation of the data set that it represents as strong or weak.</p> <p>Given a scatter plot, describe the correlation of the data set as strong or weak.</p>	<p>Define the correlation between two variables as the association and provide an example.</p> <p>Define causation between two variables as a cause and effect relationship and provide an example.</p>	<p>Explain why a strong correlation does not imply causation.</p> <p>Distinguish variables that are correlated because one is a cause of another and justify their reasoning.</p>	<p>Critique and recognize misinterpretation of correlation as causation in worked examples and explain why the reasoning is incorrect.</p> <p>Create real-world data points that suggest a strong linear correlation between two variables, but they are obviously not linked.</p>

**Instructional Focus Statements**

**Level 3:**

One of the most common misconceptions when learning statistics is assuming that a strong correlation implies causation. As teachers teach for deep understanding with standard A1.S.ID.C.6, an emphasis should be placed on interpreting a strong correlation appropriately (standard A1.S.ID.C.7). Therefore, these two standards can be paired together nicely within instruction. Students should understand correlation as the strength of association of two variables, which does not mean that changes in one variable causes changes in the other. Teachers should provide examples of contextual situations in which there exists a strong correlation, but the implication of causation is obviously incorrect. For example, teachers can show students a scatterplot of children's shoe sizes versus their vocabulary level, indicating a strong correlation between the two. Students should realize that this does not indicate that shoe size influences a child's vocabulary. In fact, students should list other factors or lurking variables that may have been affecting vocabulary instead. Until all other factors can be eliminated, we cannot imply that shoe size causes changes in vocabulary

Although students should not imply that correlation means causation, there are situations in which variables are correlated because one is the cause of

another. For example, there is a correlation between number of hours studied for a test and the grade you receive on a test because there is a cause and effect relationship between the two. So, a causal relationship between two variables implies the variables will be correlated, but a strong correlation does not imply causation.

**Level 4:**

As students develop a deep understanding of the relationship between correlation and causation, they should be given opportunities to critique the work of others and justify why their reasoning is correct or incorrect. Students should recognize when a worked example is implying correlation from causation or causation from correlation. Therefore, teachers should provide students with both types of situations. The worked examples can come from other students as they create real-world data points that suggest a strong linear correlation between two variables, but they are obviously not linked. Experiencing these situations solidifies the idea that correlation does not imply causation.