# Integrated Mathematics II 3133

**Course description:**

This course is the second of three courses in a series that uses a more integrated approach to cover the same algebra and geometry concepts and skills that are included in the traditional three course series. The problem situations, models, and technology used will foster connections among the various strands of mathematics and develop concepts from multiple perspectives.

**Standard 1.0: Number and Operations**

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

**Learning Expectations:**

The student will

1.1 demonstrate an understanding of the elements, properties and operations of real numbers;

1.2 demonstrate an understanding of the relative size of rational and irrational numbers;

1.3 connect physical, graphical, verbal, and symbolic representations of real numbers;

1.4 articulate, model and apply the concept of inverse (powers and roots);

1.5 demonstrate an understanding of absolute value;

1.6 recognize the existence of imaginary numbers.

1.7 select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;

1.8 apply matrix operations to solve real-world problems, using appropriate technology.

**Student Performance Indicators:**

**At Level 1, the student is able to**

approximate pi given a table of values for the circumference and diameter of circles;

order a set of rational and irrational numbers;

find an integral power of a positive rational number (exponents 1-3).

**At Level 2, the student is able to**

use absolute value to express the distance between two points on a number line and vice versa;

simplify a radical (radicand less than 1000);

match a given irrational number to the appropriate point on a number line and vice versa (e.g., $\sqrt{2}$, $\sqrt{30}$, pi).

**At Level 3, the student is able to**

use radicals and decimal approximations of irrational numbers to indicate calculated lengths or distances;

represent irrational numbers as lengths of lines in the coordinate plane (e.g. ,$\sqrt{5}$ is the length of the diagonal of a rectangle with base 1 and height 2).

**Sample Task:**

Students will compute the hypotenuse of a given right triangle and arrange themselves in order from smallest to largest.

**Linkages:**

Mathematics: Estimation, Measurement, and Algebra.

**Standard 2.0: Algebra**

Students will recognize, extend, create, and analyze a variety of geometric, spatial, and numerical patterns; solve real-world problems related to algebra and geometry; and use properties of various geometric figures to analyze and solve problems.

**Learning Expectations:**

The student will:

2.1 solve systems of three equations and three unknowns using a variety of techniques including inverse matrices with technology;

2.2 describe the domain and range of a function;

2.3 represent real-world problems involving sets, their intersections, union, and complements using Venn diagrams;

2.4 apply Venn diagrams in problem solving;

2.5 solve quadratic equations and inequalities using appropriate methods;

2.6 solve radical equations using appropriate methods;

2.7 graph absolute value functions and quadratic functions with emphasis on transformations;

2.8 solve real-world problems modeled by absolute value or quadratic functions;

2.9 recognize the conic sections from given information;

2.10 recognize, extend, and create numerical, geometric, and spatial patterns;

2.11 generalize patterns verbally and symbolically using function notation.

**At Level 1, the student is able to**

extend or find missing element(s) in a geometric patterns and situations (e.g., Fibonacci sequence and Golden Ratio);

solve multistep linear equations to find length, width, perimeter, and area of geometric figures;

apply the concept of rate of change to solve a real-world problem given a pattern of data;

determine the slope given a graph of a linear equation and vice versa;

determine the distance, midpoint, or slope when given the coordinates of two points (answers must be given as decimals to the nearest hundredth).

**At Level 2, the student is able to**

determine the equation of a line parallel or perpendicular to a given line, from given information (e.g., equations of lines, graphs of lines, or two points);

apply ratio and proportion to solve real-world problems involving polygons, (e.g., scale drawings, similar figures);

apply the triangle inequality property to determine which sets of side lengths determine a triangle;

determine the perimeter, area, or volume given the ratio of two similar polygons or rectangular solids;

apply the Triangle Sum Theorem or Exterior Angle Theorem to determine the measures of the angles of a given triangle with the angle measures expressed algebraically.

**At Level 3, the student is able to**

determine the equation of a circle given coordinates or the graph of the circle (e.g., the center, the endpoints of the diameter);

use manipulatives to determine relationships between linear, square, or cubic measures when one of the measures of the object has changed and represent algebraically.

apply the line of best fit given real-world data from geometric figures using technology (e.g., finding the interior angle sum of polygons when given the number of sides; find the circumference of circles when given the diameter).

recognize complete and incomplete networks;

graph plane figures on a coordinate plane and solve problems algebraically.

**Sample Task:**

Students construct designs using basic geometric constructions. Then they transfer the design to a piece of 8""X 11""pane of plexiglass and paint the pane to create a "stained glass."

**Linkages:**

Mosaic Tiling.

**Standard 3.0: Geometry**

Students will investigate, model, and apply geometric properties and relationships and use indirect reasoning to make conjectures; deductive reasoning to draw conclusions; and both inductive and deductive reasoning to establish the truth of statements.

**Learning Expectations:**

The student will:

3.1 demonstrate an understanding of geometric transformations (i.e. reflection, translation, rotation, and dilation);

3.2 apply deductive reasoning using postulates and theorems to prove conclusions from given hypotheses;

3.3 determine the truth of an implication, its converse, inverse, and contrapositive;

3.4 apply right triangle properties, including geometric mean, The Pythagorean Theorem, special right triangles, and the trigonometric ratios;

3.5 derive the distance formula for the distance between two points in a rectangular coordinate system;

3.6 apply concepts related to similar and congruent triangles;

3.7 apply properties of circles, arcs, chords, tangents, or secants to solve problems;

3.8 apply the distance and midpoint formulas in solving problems;

3.9 solve real-world problems involving area with two- and three- dimensional shapes;

3.10 use coordinates to describe position in two and three dimensions.

**Student Performance Indicators:**

**At Level 1, the student is able to**

identify corresponding parts of similar and congruent geometric figures given a diagram;

determine the length of a missing side in a right triangle when given two sides (answers must be given as simplified radicals).

**At Level 2, the student is able to**

identify properties of plane figures from information given in a diagram;

identify chords, inscribed angles, or central angles of circles given a diagram;

determine congruence or similarity relations between triangles or quadrilaterals given a diagram;

determine whether a plane figure has been translated, dilated, reflected, or rotated given a diagram and vice versa;

solve problems involving complementary, supplementary, congruent, vertical, or adjacent angles given angle measures expressed algebraically;

determine the trigonometric ratio for a right triangle needed to solve a real-world problem given a diagram;

find a missing side length in a 30-60-90 or 45-45-90 degree triangle without rationalizing the denominator

apply properties of quadrilaterals to solve a real-world problem given a diagram (opposite sides and angles, consecutive sides and angles, or diagonals);

solve real-world problems involving measures of interior or exterior angles of regular polygons;

identify the appropriate segment of a triangle given a diagram and vice versa (i.e. median, altitude, angle bisector, perpendicular bisector);

determine which three-dimensional solid is represented by a given net and vice versa (two-dimensional drawing);

determine the area of indicated regions involving circles, squares, rectangles, and/or triangles;

justify triangle congruence given a diagram (i.e., ASA, SSS, AAS, SAS, or Hypotenuse/ Leg);

determine if a triangle is a right triangle given the length of all the sides of a triangle.

investigate and apply the properties of angles, arcs, chords, tangents, and/or secants using technology or manipulatives; find the area of a sector of a circle given a diagram.

use inductive and deductive reasoning to make conjectures, draw conclusions, and solve problems;

recognize and articulate relationships among families of geometric figures (e.g., quadrilaterals, prisms).

**At Level 3, the student is able to**

use coordinates to communicate the location of a three-dimensional figure that has been rotated or reflected;

write and defend indirect and direct proofs;

use logical reasoning to solve problems in the real world;

use manipulatives to explore the geometric mean of similar triangles;

use appropriate tools or technology to develop geometric and spatial concepts;

construct three-dimensional objects using physical materials and manipulatives;

compare and construct quadrilateral properties using a variety of models (e.g., Venn diagrams, family trees, manipulative mobiles).

**Sample Task:**

Students construct and use a hypsometer to measure several tall structures on the school grounds.

**Linkages:**

Mathematics: Measurement. Surveying and Art.

**Standard 4.0: Measurement**

Students will apply appropriate units of measurement; develop effective estimation and computation strategies for solving real world problems involving length, area, and volume; and choose appropriate techniques and tools to measure quantities in order to meet specifications for precision, accuracy, and tolerance.

**Learning Expectations:**

The student will:

4.1 choose appropriate techniques and tools to measure quantities in order to meet specifications for tolerance;

4.2 perform operations on algebraic expression and informally justify the procedures chosen;

4.3 use concepts of length, area, and volume to estimate and solve real-world problems;

4.4 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;

4.5 use estimation to make predictions and determine reasonableness of results;

4.6 demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hr, revolutions per minute, cost per unit);

4.7 apply geometric properties in constructions using a variety of tools (e.g. paper folding, geometric software, reflections tools).

**Student Performance Indicators:**

**At Level 1, the student is able to**

determine the perimeter or area of a triangle or rectangle when the dimensions are given as first degree binomials in one variable;

determine the measure of an angle using a protractor.

solve real world problems involving perimeter or area of three or four sided plane figures.

**At Level 2, the student is able to**

determine the volume or surface area of a rectangular solid or cylinder in a real-world situation;

construct bisectors of angles and line segments, perpendicular lines, congruent line segments and angles, and perpendicular bisectors using a variety of methods (e.g., patty paper, technology).

**At Level 3, the student is able to**

determine whether a reading falls within an acceptable tolerance range.

choose appropriate techniques and tools to measure quantities in order to meet specification for precision, accuracy, and tolerance;

locate the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ on a number line by using the Pythagorean relationship and a straightedge and compass, manipulatives, or technology;

solve problems involving surface area of pyramids, cones, and spheres.

**Sample Task:**

Students construct designs using basic geometric constructions. Then they transfer the design to a piece of 8""X 11""pane of plexiglass and paint the pane to create a "stained glass." Students construct one of the regular 3-dimensional solid and compute the volume and surface area.

**Linkages:**

Mathematics – Geometry and Number & Operations. Surveying, construction, and architecture. .Mosaic Tiling.

**Standard 5.0: Data Analysis and Probability**

Students will investigate, explore, and apply geometric representations to calculate theoretical probability; and will use data from geometric figures to investigate relationships.

**Learning Expectations:**

The student will:

5.1 demonstrate an understanding of different sampling methods and when each is appropriate;

5.2 use simulations to demonstrate probability experiments;

5.3 use a variety of techniques to determine equations of best fit for quadratic data sets;

5.4 analyze the validity of statistical conclusions;

5.5 determine the probability of an event;

5.6 determine the probability of mutually exclusive events.

**Student Performance Indicators:**

**At Level 1, the student is able to**

make a prediction from a geometric representation of a real-world data set;

**At Level 2, the student is able to**

determine the probability of an event represented as a subset of the area of a two-dimensional geometric figure.

collect and analyze data to make conjectures about geometric relationships.

**Sample Task:**

Construct two 1' X 1' dart boards and draw circular targets on each that are externally tangent to each adjacent circle and to the edge of the board. Draw two circles on one dartboard and three on the other. Throw randomly and count the throws that hit the board to determine which board yields the highest probability of a dart's landing in a circle. Calculate the probability for each bard.

**Linkages:**

Game theory.